Worst-Case Execution-Time Analysis

WCET Analysis

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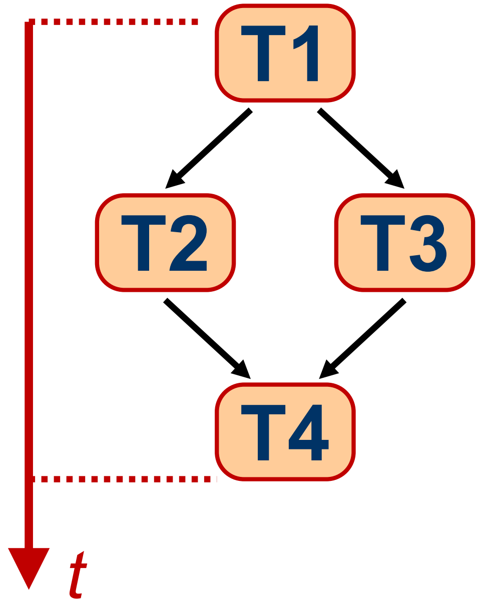
Time in RTS Construction

Design
Architecture, resource planning, schedules

Implementation

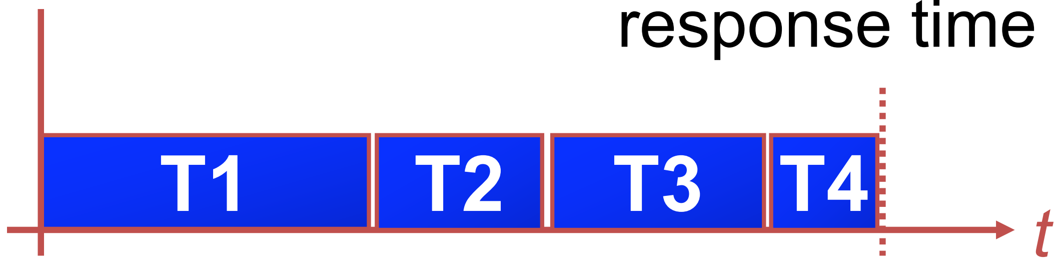
Timing Analysis
Schedulability analysis, WCET analysis

From Design to Implementation



Task set with precedence constraints and deadline

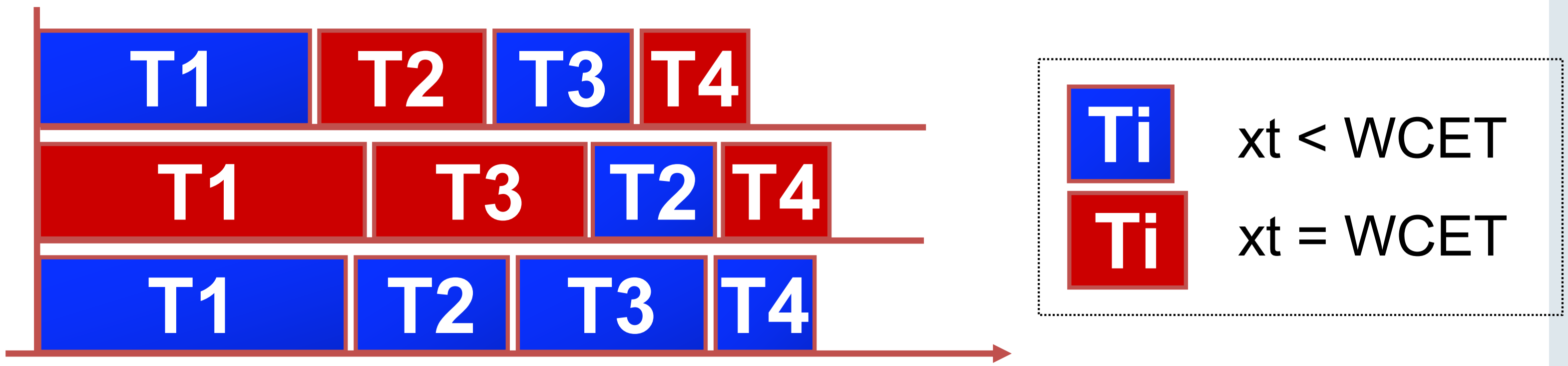
Task sequence: execution times, response time



Can we guarantee that: response time < deadline?

Timing-Analysis Abstraction

In general it is infeasible to model all possible execution scenarios and combinations of task execution times



Timing analysis abstraction of different execution times: one single value ⇨ WCET (worst-case execution time)

RTS Timing Analysis

Schedulability objects

- Units of execution (simple tasks) with WCET
- Precedence relations
- Synchronization, communication, mutual exclusion
- Priorities

WCET-analysis objects

- Simple tasks

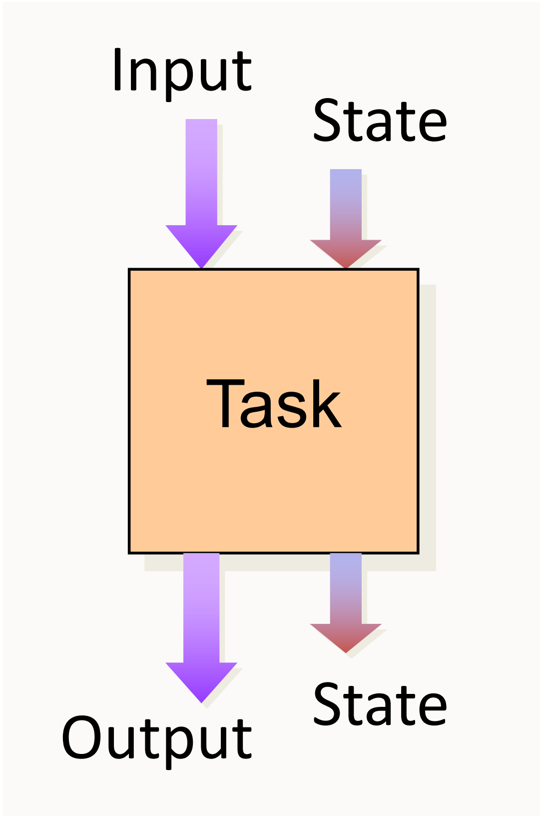
Interference ... (nasty and therefore widely neglected)

- “external” changes of task state that influence exec. time

Simple Task

- Inputs available at start
- Outputs ready at the end
- No blocking inside
- No synchronization or communication inside
- Execution time variations only due to differences in
  - inputs
  - task state at start time

  (no external disturbances)



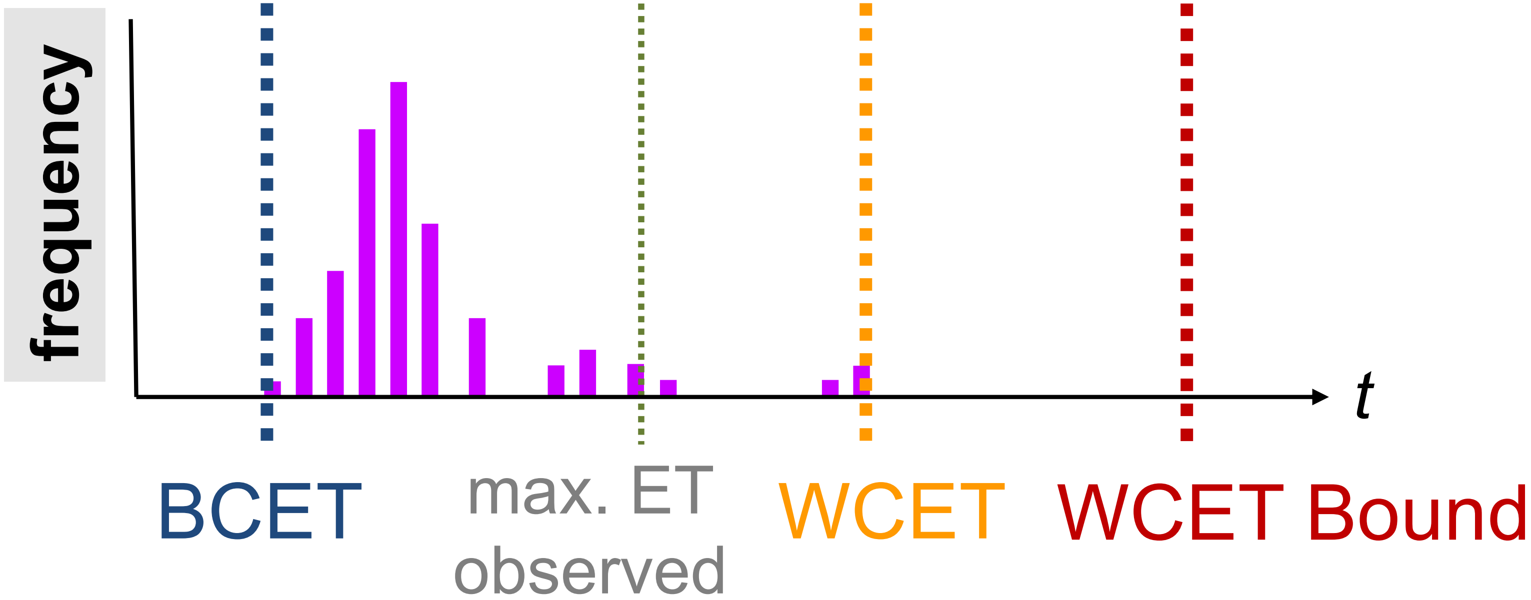
Worst-Case Execution Time

Def. Worst Case Execution Time (WCET):

WCET of software is the maximum time it takes to execute

- a given piece of code
- in a given application context (inputs, state)
- on a given machine

Task-Timing Terms



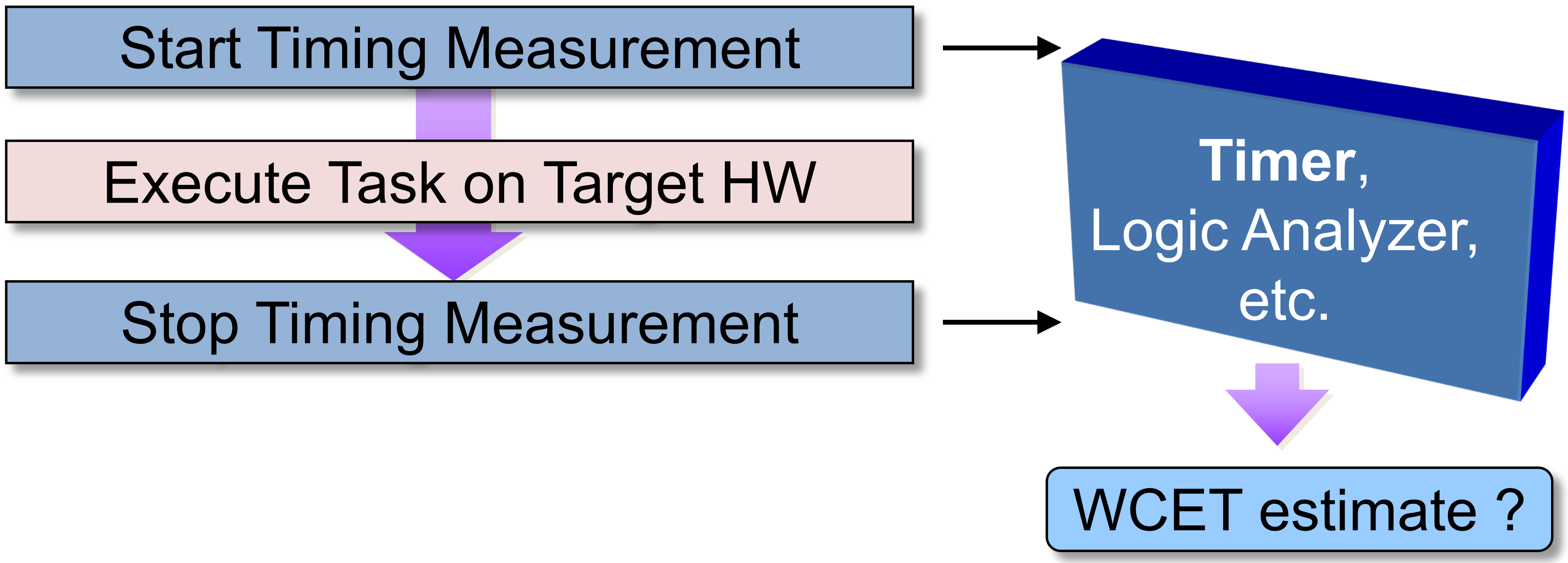
BCET … best-case execution time
WCET … worst-case execution time

WCET Analysis

WCET Analysis goal: derive upper bounds for the execution time of pieces of code

- ➪ WCET bounds must be safe
  (i.e., must never underestimate the WCET)
- ➪ WCET bounds should be tight
  (i.e., must not be too pessimistic)
- ➪ The analysis cost should be reasonable
  (i.e., computational efforts must not be too high)

Measuring WCET



Why not just Measure WCET?

- Measuring all different traces is intractable (e.g., 10^{40} different paths in a mid-size task)
- Selected test data for measurement may fail to trigger the longest execution trace

a) Test-data generation: rare execution scenarios may be missed (e.g., exception handling, …)

b) Internal processor state may not have been in its worst-case setting at the beginning

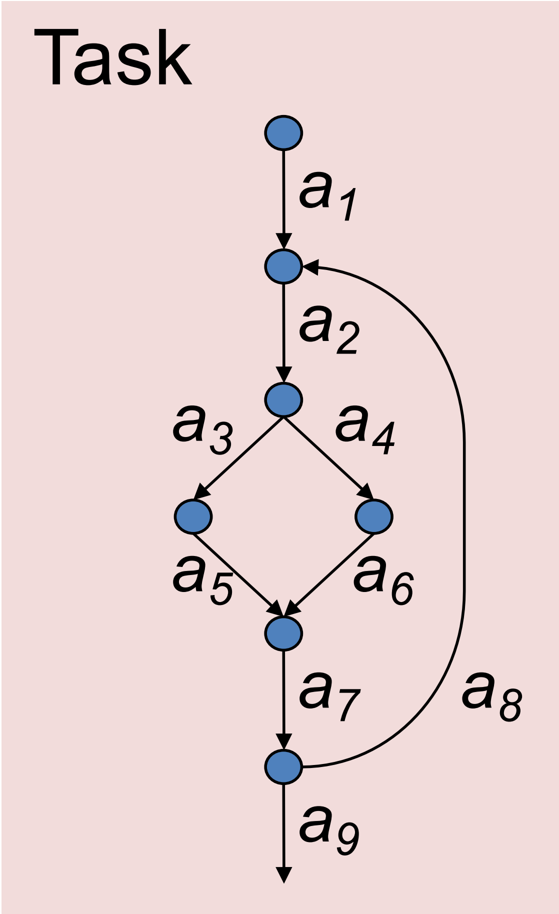
Measurements: rough WCET estimates, WCET testing

Static WCET Analysis

Static WCET Analysis: computes upper bounds for the execution time of pieces of code

- models software, hardware, and context
  - SW: source code, executable (with addresses resolved)
  - HW: processor (pipeline), memory (areas, caches), …
  - Context: Initial software + hardware state

WCET Determinants

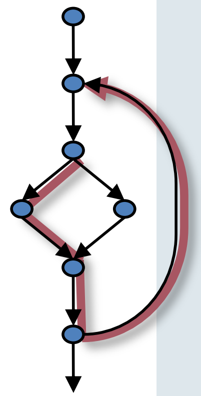


- Possible sequences of actions of the task (= execution paths) in given application
- The duration of each occurrence of an action on each possible (= feasible) path

WCET Determinants

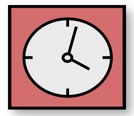
Sequences of actions are determined by

- Semantics of code (incl. hardware specific semantics, implementation specifics)
- Possible inputs in context (appl., call context)



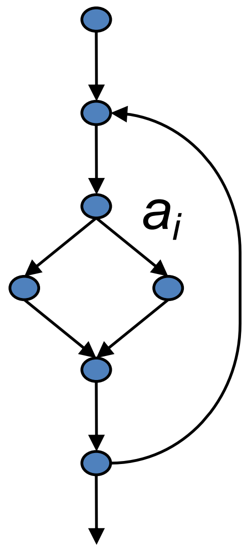
Duration of actions

- Implementation of instructions in HW
- HW state that influences timing (caches, pipelines, etc.)
  - task-internal effects
  - external effects ⇨ start state; state after preemption



Path Timing – Simple vs. Complex Arch.

Execution time of path *k*: $xt(p_k)$



<u>Simple Architecture</u>

Duration of each action a_i is constant:

$$xt(p_k) = \sum_i n_{k,i}\, t(a_i)$$

<u>Complex Architecture</u>

Durations of actions vary:

$$xt(p_k) = \sum_i \sum_{j(k)} t(a_{i,j(k)})$$

Reasons: pipelining, caches, parallelism in CPU, …

WCET Analysis – The Challenges

Path analysis: identifying (in)feasible paths

- Syntactic restrictions
- Semantic restrictions
- Input-data space

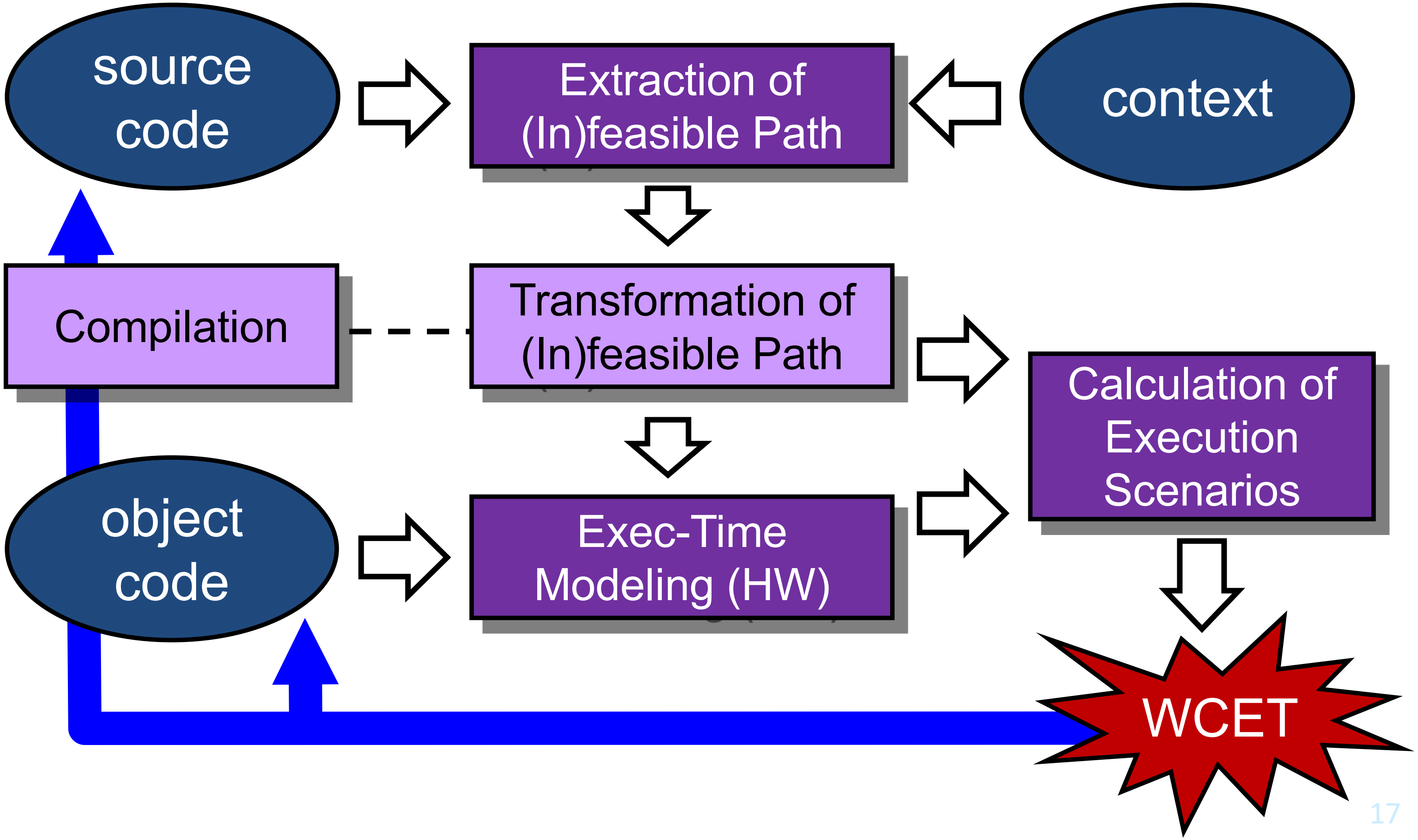
Modelling of hardware timing

WCET calculation

Dealing with different levels of code representation

- Source-language user interface versus
- Execution-time modeling at machine-code level

Generic WCET Analysis Framework



Path Information (= Flow Facts)

Loop bounds have to be known

Description of further characteristics improves the quality of WCET analysis

for i := 1 **to** N **do** ←	loop bound: N
for j := 1 **to** i **do** ←	loop bound: N; local: i: $1..N$

```
for i := 1 to N do
    for j := 1 to i do
    begin
        if c1 then A.long
               else B.short
        if c2 then C.short
               else D.long
    end
```

Loop body (begin … end): $\frac{(N+1)N}{2}$ executions

Path Information of Interest

Simple Architectures

- Information how often actions occur
- ➩ Execution-fequency bounds and relations
- ➩ Notation: marker, relations, and scopes

Complex Architectures

- Information about occurrence order / patterns
- ➩ Characterization of (im)possible paths
- ➩ Notation: based on regular expressions, IDL (path Information Description Language)

Realization of Path Analysis

In general, automation is impossible (theoretically equivalent to halting problem; state space …)

Some information can be extracted automatically

- abstract interpretation
- symbolic modeling
- simulation

⇨ Program constructs, annotations,
interactive input of path constraints by the user
(≈ documentation of possible execution traces)

Markers, Relations and Scopes

```
SCOPE
{
    for (i=0; i<N; i++)
    {
        MAX_ITERATIONS(N);
        for (j=0; j<i; j++)
        {
            MAX_ITERATIONS(N);
            MARKER(M1);
                …
        }
    }
    REL(FREQ(M1) == N * (N+1) / 2);
}
```

WCET Calculation Techniques

- Tree-based WCET calculation
- (Path-based WCET calculation)
- WCET analysis based on implicit path enumeration (IPET)

Tree-Based WCET Calculation

- Also called "timing-schema approach"
- Bottom-up traversal of syntax tree
- Timing schema: Rule computes the timing of a syntactic unit from its constituents.

Tree-Based WCET Calculation

```
for (i=0; i<N; i++)
{
  …
}
```

T(for) =
(LB+1)*T(test) + LB*T(body)

LB … loop bound

```
if (a==5)
{
  …
}
else
{
  …
}
```

T(if) =
T(test) +
max(T(then), T(else))

WCET Calculation using IPET

IPET ... Implicit Path Enumeration Technique

Program given as control-flow graph (CFG).

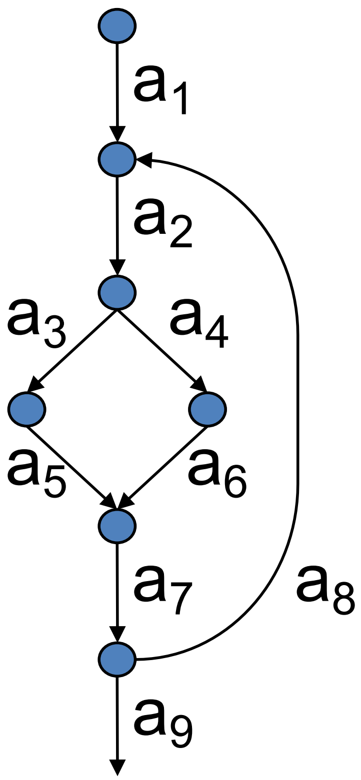
Use methods like integer linear programming (ILP) or constraint-solving to calculate a WCET bound.

WCET analysis as optimization/maximization problem:

- Maximize goal function describing execution time under
- a set of constraints describing possible paths;
  Constraints characterize:
  - the structure of the control-flow graph,
  - control-flow limitations due to semantics, and
  - context.

WCET IPET: goal function (simple HW)

Program



WCET: maximize $\sum x_i \cdot t_i$

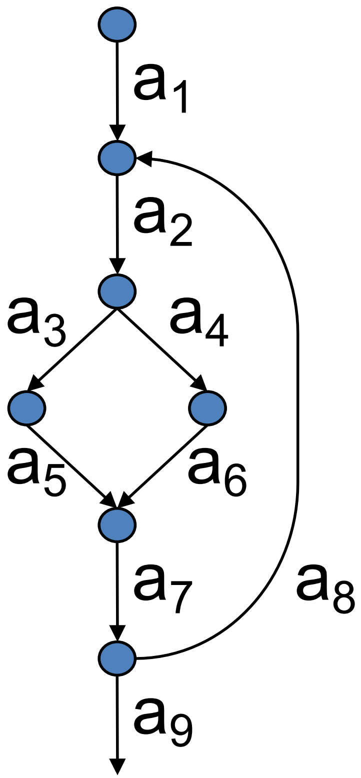
- x_i … variable: execution frequency of CFG edge a_i
- t_i … coefficient: execution time of edge a_i

Example: t_1: 40, t_2: 56, t_3: 82, t_4: 12, t_5: 10, t_6: 10, t_7: 32, t_8: 10, t_9: 102

Goal function: $40x_1 + 56x_2 + 82x_3 + 12x_4 + 10x_5 + 10x_6 + 32x_7 + 10x_8 + 102x_9$

WCET IPET: constraints (simple HW)

Program



Flow constraints:

$x_1 = 1$

$x_1 + x_8 = x_2$

$x_2 = x_3 + x_4$

$x_3 = x_5$

$x_4 = x_6$

$x_5 + x_6 = x_7$

$x_7 = x_8 + x_9$

$x_2 <= LB * x_1$

Example: loop bound 20

Loop constraint: $x_2 <= 20 * x_1$

WCET Calculation using IPET

IPET solution = WCET bound

Variable values (x_i) characterize worst-case execution path(s)

Advantages:

Description of complex flow facts is possible.

Generation of structural constraints is simple.

Optimization problem can be solved by existing tools.

Drawbacks:

Solving ILP is in general NP hard → tool runtime.

Flow facts that describe execution order are difficult to integrate.

Exec-Time Modeling for Complex HW

Maps a sequence of instructions to an execution time.

Execution time of instruction may vary due to:

- different values of input parameters;
  (max. value documented in HW manuals)
- internal state of the processor;
  (footprint of the execution history)

HW features that influence the processor state:
instruction & data cache, instruction parallelism, branch prediction, speculative execution, …

Exec-Time Modeling (2)

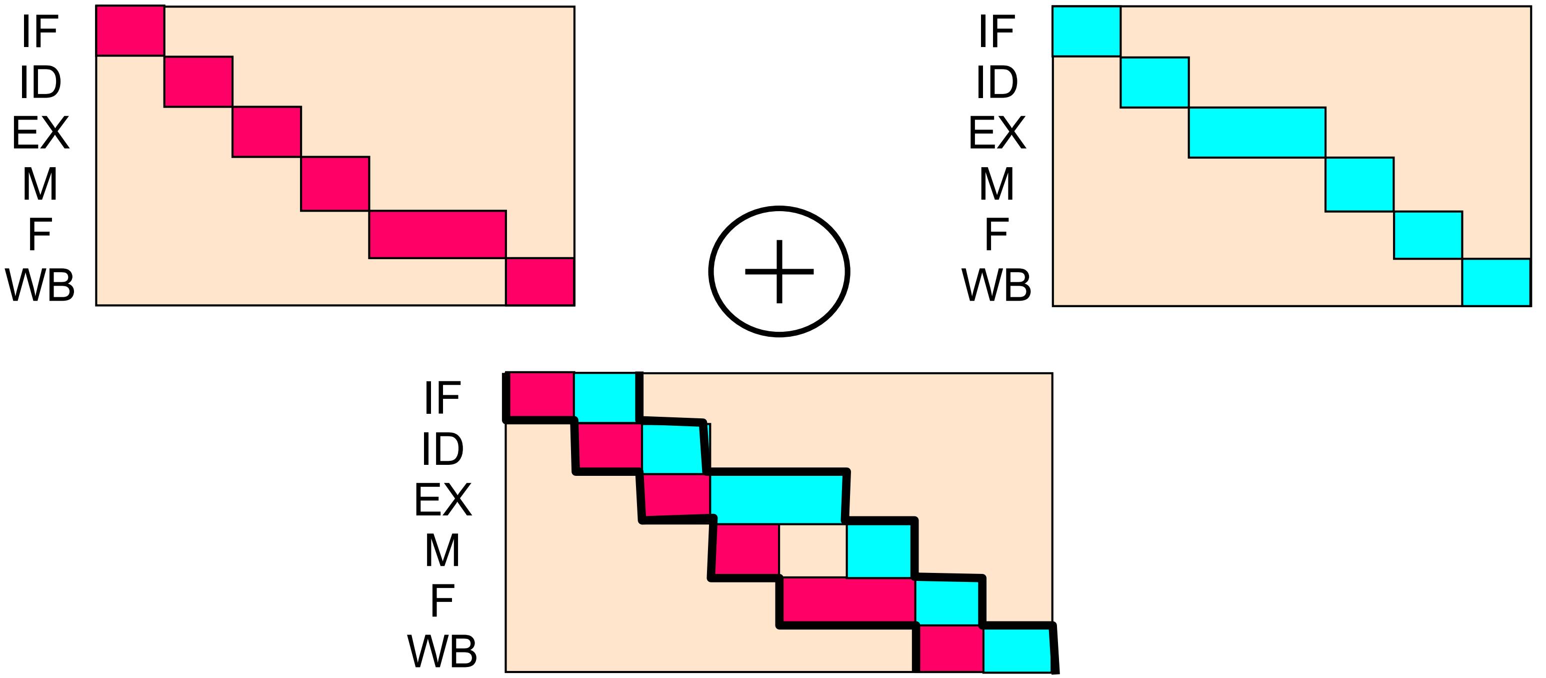
Exec-time modeling typically done before WCET calculation in separate phases:

1. cache analysis
2. pipeline analysis
3. path analysis + WCET calculation

Modeling Pipelines (Example)

Basic operations on reservation tables:

Sequential combination of two reservation tables



Caches and WCET Analysis

Purpose: Bridge gap between fast CPU and slow memory

Essential to analyze caches on many architectures

Example: 40 cycles for a miss on MPC755

Types of Caches: Instructions, Data, BTB, TLB

Design: Direct Mapped, Set/Fully Associative

Replacement Policy: LRU, FIFO, PLRU, PRR

Many varieties: read-only / write through / write back, write (no) allocate, Multi-Level Caches (inclusive/exclusive), ...

WCET analysis: assuming that every memory access is a cache miss yields too pessimistic results

Categories of Cache Behavior

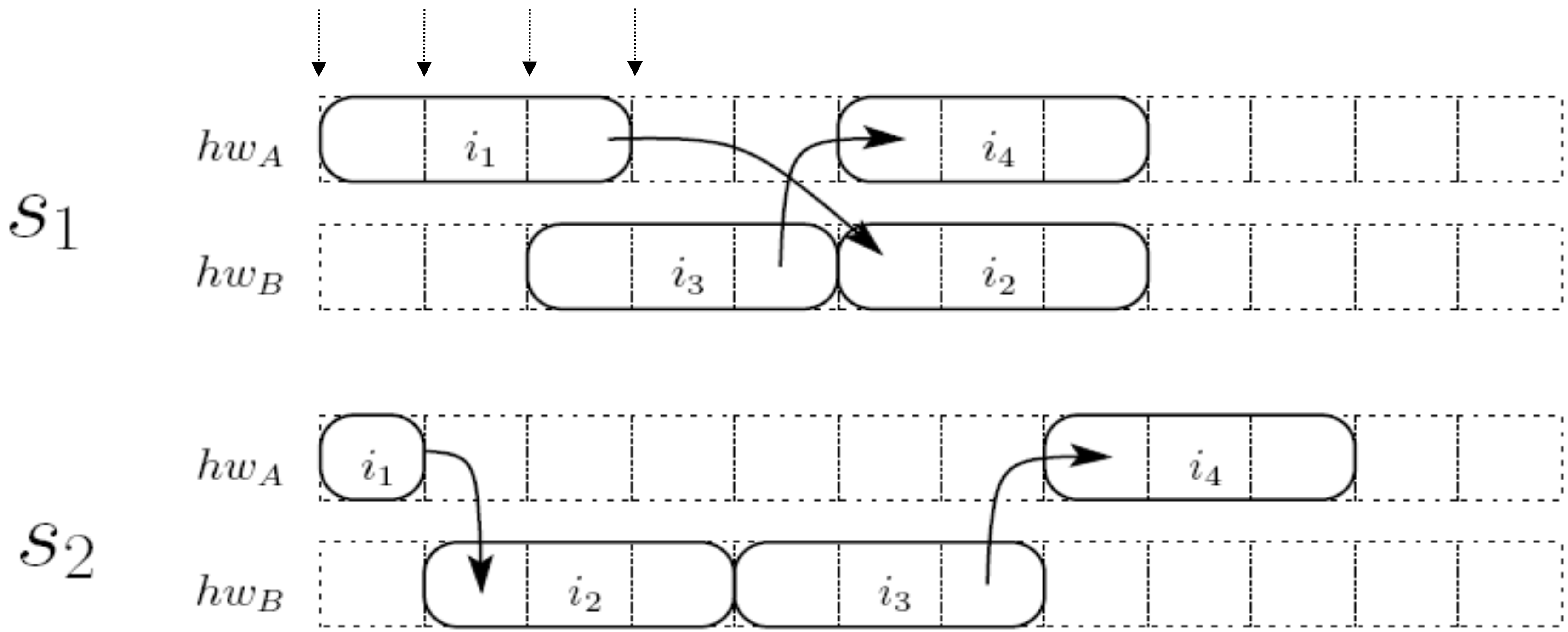
The cache behavior is analyzed to model the different timing of memory accesses – fast cache hits vs. slow cache misses

Categorization of memory accesses:

ah	always hit	each access to the cache is a hit (MUST analysis)
am	always miss	each access to the cache is a miss (complement of MAY analysis)
ps(S)	persistent	for each entering of context S, first access is **nc**, but all other accesses are hits (PERSISTANCE analysis)
nc	not classified	the access is not classified as one of the above categorizations

Timing Anomalies (Example)

- Discrepancy between local and global timing
- Makes divide-and-conquer analysis difficult



Summary

Timing analysis

- Scheduling/schedulability – WCET analysis – interferences

WCET definition

- Simple tasks: code; machine; context (application, situation)

Measuring versus static WCET analysis

WCET framework

- Path analysis
- Modeling of hardware (instruction & memory-access timing)
- WCET computation technique