Einführung in Artificial Intelligence SS 2024, 4.0 VU, 192.027

Exercise Sheet 4 – Knowledge Representation and CSP

For the discussion part of this exercise, mark and upload your solved exercises in **TUWEL** until Wednesday, June 5, 23:55 CEST. The registration for a solution discussion ends on Friday, June 7, 23:55 CEST. Be sure that you tick only those exercises that you can solve and explain!

In the discussion, you will be asked questions about your solutions of examples you checked. The discussion will be evaluated with 0–25 points, which are weighted with the fraction of checked examples and rounded to the next integer. There is *no minimum number of points* needed for a positive grade (i.e., you do not need to participate for a positive grade, but you can get at most ≈80% without doing exercises).
Note, however, that *your registration is binding*. Thus, *if* you register for a solution discussion, then it is *mandatory* to show up. Not coming to the discussion after registration will lead to a reduction of examination attempts from 4 to 2.
Please ask questions in the **TUWEL** forum or visit our tutors during the tutor hours (see **TUWEL**).

Exercise 4.1: A magic square is an $N \times N$ square grid, containing each number from 1 to N^2 exactly once, where the sum of each row, column, and diagonal must be the same. We consider a 4×4 grid in which the sums should all be 34:

8	A	14	B
13	C	7	D
E	16	F	6
G	5	H	I

Some of the numbers are already given, the unknown numbers have been replaced by variables.

(a) Describe the corresponding CSP with its variables and constraints and specify the initial domain of each variable.

(b) Draw the constraint graph.

(c) Find a solution of the puzzle. How many unique solutions exist? You may use automated solvers or program your own, but be sure that you can explain how a solution could be found by hand.

Exercise 4.2:

(a) Given a single ternary constraint $A + B = C$. Transform this constraint into 3 binary constraints achieving the same functionality using auxiliary variables.

(b) Show how unary constraints can be eliminated by altering the domains of variables.

Exercise 4.3: The Joker, the Penguin, and the Riddler meet. They make the following statements:

- JOKER: Penguin tells the truth.

- PENGUIN: Riddler lies.
- RIDDLER: You two either both tell the truth or both lie.

Which of the villains lie and which tell the truth?

To answer this puzzle, represent the statements in propositional logic and use truth tables to determine a model of the conjunction of the three statements. Use the atomic formulas J, P, and R for representing the statements that

- the Joker tells the truth,
- the Penguin tells the truth, and
- the Riddler tells the truth,

respectively.

Exercise 4.4: Prove the following semantic consequences:

- $A, B \models A \wedge B$
- $A, A \vee B \not\models B$
- $A \vee B, A \rightarrow C, B \rightarrow D \models C \vee D$

Exercise 4.5: Consider the following sentences:

1. Every student knows a student who passed the exam.
2. Every student who solved all the exercises also passed the exam.
3. Some student did not solve the exercises, but passed the exam.
4. Some student who passed the exam does not know any student who solved all of the exercises.

(a) Formalise these sentences in first-order logic by using the following predicates:

- $K(x, y)$: "Student x knows student y",
- $P(x)$: "Student x passed the exam",
- $E(x)$: "Student x solved all of the exercises",

(b) Is the set of formulas resulting from the formalisation in (a) satisfiable? If yes, provide a model; if no, give a justification.

Exercise 4.6: Prove the unsatisfiability of the following propositional formula using resolution:

$$(\neg x \vee \neg y \vee \neg z) \wedge (\neg x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge$$
$$(\neg x \vee \neg y \vee z) \wedge (x \vee \neg y \vee \neg z) \wedge (x \vee y \vee \neg z) \wedge z.$$