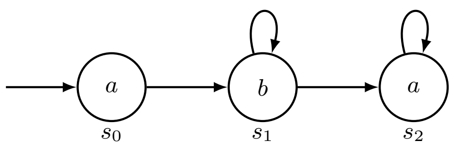
VU Programm- und Systemverifikation
Homework: Temporal Logic and Automated Reasoning

Due date: May 27, 2021, 1pm

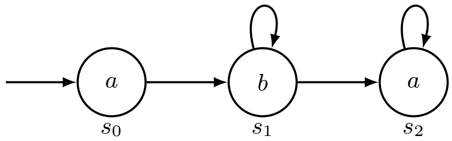
Task 1 (5 points): Consider the following Kripke Structure:



For each formula, give the states of the Kripke structure for which the formula holds. In other words, for each of the states from the set $\{s_0, s_1, s_2\}$, consider the computation trees starting at that state, and for each tree, check whether the given formula holds on it or not.

1. $\mathbf{A}\,\mathbf{F}\,\mathbf{G}\,a$: $\{s_2\}$
2. $\mathbf{A}\,\mathbf{F}\,\mathbf{A}\,\mathbf{G}\,b$: $\emptyset$
3. $\mathbf{A}\,(a \wedge \mathbf{X}\,b)$: $\{s_0\}$
4. $\mathbf{A}\,(b\,\mathbf{U}\,a)$: $\{s_0, s_2\}$
5. $\mathbf{A}\,\mathbf{X}\,\mathbf{X}\,a$: $\{s_2\}$
6. $\mathbf{A}\,\mathbf{G}\,\mathbf{F}\,a$: $\{s_2\}$
7. $\mathbf{A}\,\mathbf{G}\,\mathbf{F}\,b$: $\emptyset$
8. $\mathbf{EG}\,\mathbf{F}\,b$: $\{s_0, s_1\}$
9. $\mathbf{A}\,(a\,\mathbf{U}\,b)$: $\{s0, s_1\}$
10. $\mathbf{E}(b\,\mathbf{U}\,a)$: $\{s_0, s_1, s_2\}$

Task 2 (3 points): Consider the following Kripke Structure with initial state s_0:



1. Does the LTL formula $\mathbf{A}\,\mathbf{F}\,\mathbf{G}\,b$ hold in the initial state s_0?

☐ yes ☑ no

There is at least one path ($s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow s_2 \cdots$) on which b does not hold globally from some point on.

2. Does the CTL formula $\mathbf{A}\,\mathbf{F}\,\mathbf{EG}\,b$ hold in the initial state s_0?

☑ yes ☐ no

First note that $\mathbf{EG}\,b$ holds in state s_1 (there *exists* an outgoing path along with b holds indefinitely). Also, if we start in s_0, we necessarily reach s_1 after one step, so it is *eventually* reached on every path. Hence the formula holds.

3. Do the formulas (i) and (ii) above express the same property?

☐ yes ☑ no

$\mathbf{A}\,\mathbf{F}\,\mathbf{EG}\,b$ holds on the Kripke structure above. At the same time, $\mathbf{A}\,\mathbf{F}\,\mathbf{G}\,b$ doesn't hold on the same Kripke structure. Consequently, the given Kripke structure is a counterexample to the equivalence of both temporal logic formulas.

Task 3 (2 points): Encode the following statements in temporal logic (LTL if possible, CTL otherwise):

1. Whenever the execution of a program encounters a *fault* it is possible to *recover* eventually.

$$\mathbf{A}\,\mathbf{G}\,(\mathit{fault} \Rightarrow \mathbf{EF}\,\mathit{recover})$$

2. If an execution encounters *fault*s in three subsequent states, it will never *recover* again.

$$\mathbf{G}\,((\mathit{fault} \wedge \mathbf{X}\,\mathit{fault} \wedge \mathbf{X}\,\mathbf{X}\,\mathit{fault}) \Rightarrow \mathbf{G}\,\neg\mathit{recover})$$

3. Whenever a state labeled with a is reached, a state labeled with b will be reached at a *strictly later* point.

$$\mathbf{G}\,(a \Rightarrow \mathbf{X}\,\mathbf{F}\,b)$$

4. All paths starting at the initial state lead to a cycle that does not contain a state labeled a.

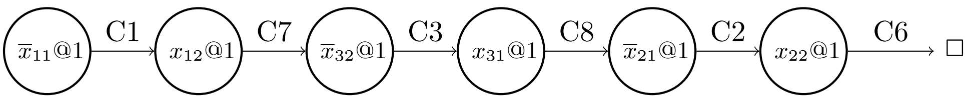
$$\mathbf{F}\,\mathbf{G}\,\neg a$$

Task 4 (5 points): Consider the following formula in propositional logic; is it satisfiable? If yes, provide a satisfying assignment, if not, give the reasoning that leads to this conclusion.

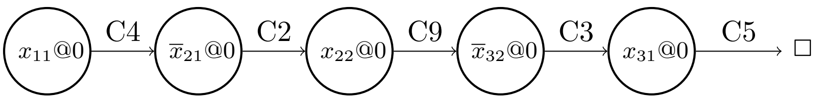
$$\overbrace{(x_{11} \vee x_{12})}^{C1} \wedge \overbrace{(x_{21} \vee x_{22})}^{C2} \wedge \overbrace{(x_{31} \vee x_{32})}^{C3} \wedge \underbrace{(\neg x_{11} \vee \neg x_{21})}_{C4} \wedge \underbrace{(\neg x_{11} \vee \neg x_{31})}_{C5} \wedge \overbrace{(\neg x_{12} \vee \neg x_{22})}^{C6} \wedge \overbrace{(\neg x_{12} \vee \neg x_{32})}^{C7} \wedge \underbrace{(\neg x_{21} \vee \neg x_{31})}_{C8} \wedge \underbrace{(\neg x_{22} \vee \neg x_{32})}_{C9} \quad (1)$$

(The clauses in Formula (1) are labelled $C1$ to $C9$.)

- For each decision, provide the resulting implication/conflict graph!
- For each conflict you reach, provide a corresponding learned clause!
- Decision: $\neg x_{11}@1$



- Conflict clause: (x_{11})
  Unit clause, we revert to level 0



- Conflict at level 0, hence unsatisfiable

Task 5 (3 points):

1. Consider the following formulas in propositional logic; are they satisfiable? If yes, provide a satisfying assignment over booleans, if not, give the reasoning that leads to this conclusion.

$$(\neg a \vee \neg b) \ \wedge \ c \ \wedge \ (b \vee d) \ \wedge \ \neg d \ \wedge \ e \ \wedge \ (\neg e \vee \neg c \vee a) \tag{2}$$

   - Unsatisfiable (by unit propagation).
   - $\neg d \wedge (b \vee d) \Rightarrow b$, $b \wedge (\neg a \vee \neg b) \Rightarrow \neg a$, hence $(\neg e \vee \neg c \vee a)$ is conflicting.

$$f \ \wedge \ (\neg g \vee f) \ \wedge \ (h \vee \neg f) \ \wedge \ (g \vee h) \ \wedge \ (\neg g \vee \neg h) \tag{3}$$

   - Satisfied by $f, h, \neg g$

2. Consider the following formulas in Equality Logic; are they satisfiable? If yes, provide a satisfying assignment over integers, if not, give the reasoning based on equivalence classes that leads to this conclusion.

$$i = j \ \wedge \ j = k \ \wedge \ k = l \ \wedge \ l \neq m \ \wedge \ l \neq n \ \wedge \ m = n \ \wedge \ o \neq p \wedge o = q \tag{4}$$

   - Satisfiable.
   - assign 1 to $\{i, j, k, l\}$, assign 2 to $\{m, n\}$, assign 3 to $\{o, q\}$, assign 4 to p

$$i = j \ \wedge \ j = k \ \wedge \ k = l \ \wedge \ l \neq n \ \wedge \ m = n \ \wedge \ g(i) \neq g(m) \ \wedge \ f(i) \neq f(l) \tag{5}$$

   - Unsatisfiable
   - As $\{i, j, k, l\}$ are in the same equivalence class, $f(i) \neq f(l)$ is contradictory

3. Check the satisfiability of the following SMT formulas. Assume that $x, y \in \mathbb{Z}$ are integer constants, and $f : \mathbb{Z} \to \mathbb{Z}$ and $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ are unary and binary uninterpreted functions over integers respectively. Whenever a formula is satisfiable, give a satisfying assignment for it, i.e., integer values for all constants and function interpetations over integers that make the formula true under the assignment. Whenever a formula is not satisfiable, give a reason why it is unsatisfiable.

$$g(3, y) = 5 \wedge g(y, 3) = 4 \wedge g(y, x) = g(x, y) \tag{6}$$

- satisfiable
- $y = 0$, $x = 0$, $g(3, 0) = 5$, $g(0, 3) = 4$, $g(0, 0) = g(0, 0)$

$$x = y \wedge f(f(y)) = f(x) \wedge f(1) \neq g(1, y) \wedge 1 = f(x) \wedge g(1, x) = f(f(x)) \tag{7}$$

(a) unsatisfiable

(b) $(x = y) \Rightarrow (f(x) = f(y))$

(c) hence $f(y) = 1$ follows from $f(x) = 1$

(d) hence $f(1) = 1$ follows from $f(f(y)) = (f(x))$

(e) hence $g(1, y) \neq 1$

(f) hence $g(1, x) = f(1)$ from $g(1, x) = f(f(x))$

(g) hence $g(1, x) = 1$, contradicts (e) since $x = y$

Task 6 (2 points): Indicate whether the following statements are true or false!

Statement	**True**	**False**
Any CTL formula starting with **A** can be expressed in LTL	○	✓
CTL* is the union of all CTL and LTL formulas.	○	✓
Every CTL formula has an equivalent CTL formula containing only **AF**, **EX**, and **EU**.	✓	○
There is a non-empty Kripke structure K that satisfies $(\mathbf{AG}\,\mathbf{AF}p) \wedge (\mathbf{EF}\,\mathbf{EG}\neg p)$.	○	✓

Upload a pdf file with your solutions to TUWEL by May 27, 2021.