Project and Enterprise Financing

Exercises for Test 2

Exercise A2_1:

Suppose €1 has been invested in 1800 at 2.3% interest compounded yearly.

(a) How much would that investment be worth in 2019?

(b) What if the interest rate is 4.5% p.a.?

a) (1€ * 1.023) ^ (2019-1800) = 145.4667657€

b) (1€ * 1.045) ^ (2019 – 1800) = 15 362.70206€

Exercise A2_2:

The number of years n required for an investment at interest rate r to double in value must satisfy $(1+r)^n = 2$. Calculate n for r = 0.1% p.a., 1% p.a., 3% p.a., 7% p.a., and 10% p.a.

(1 + r) ^n = 2

➔ solve for n

$$n = \frac{\log(2)}{\log(1+\mathrm{r})}$$

n1= log(2)/log(1+0.001) = 649.49 years
n2=log(2)/log(1+0.01) = 69.66 years
n3=log(2)/log(1+0.03) = 23.45 years
n4=log(2)/log(1+0.07) = 10.24 years
n5=log(2)/log(1+0.1) = 7.27 years

Exercise A2_3:

Find the annually payed interest rate that yields the same value as the following monthly payed interest rates:

(a) 4.5% p.a. compounded monthly
(b) 12% p.a. compounded monthly

r(m,n)=m*((1+r/n)^(n/m)-1)) ... n = annual frequency old, m = ann. freq. new

a) n = 12, m = 1, r = 0,045

Input:

$$1\left(\left(1+\frac{0.045}{12}\right)^{12/1}-1\right)$$

Result:

0.04593982504059053

b) n = 12, m = 1, r = 0,12

Input:

$$1\left(\left(1+\frac{0.12}{12}\right)^{12/1}-1\right)$$

Result:

0.12682503013196972

Exercise A2_4:

An investor owns 1,000 of company XY and has in addition 25,000 EUR in cash. We can observe the following stock prices, subscription rights (SR), and dividend payments:

Date	Share price (EUR)	SR-value (EUR)	Dividend (EUR)
Thu 10.10.	95		
Fri 11.10.	96		
Mon 14.10.	78	22	
Tue 15.10.	77.5		
Wed 16.10.	76		4
Thu 17.10.	75		

Calculate:

(a) the stock value, the cash value, and the total value of the investor (a1) right after the seasoned equity offering (i.e. before trading starts on Mon, 14.10.), and (a2) at the end of Mon, 14.10. (i.e. after trading stopped), when (i) she exercises all subscription rights (i.e. buys new shares), or (ii) sells all subscription rights at the calculated fair value (i.e. at 22 EUR). New shares can be bought for 30 EUR each and 2 old shares are necessary to buy one new share (2:1 ratio).

(b) total daily returns for the investor.

a) Stock Value, Cash Value, Total Value

	(a1,i)	(a1,ii)	(a2,i)	(a2,ii)
Stock Value	111000	74000	117000	78000
Cash Value	10000	47000	10000	47000
Total Value	121000	121000	127000	125000

b) Daily Returns

SR-Value	Dividends	ft	**Returns**	Calculation
			1,053%	= (Pt - Pt-1) / Pt-1
22		**0,77083333**	5,405%	= (Pt - Pt-1) / Pt-1
			-0,641%	= (Pt - **ft** * Pt-1) / (**ft** * Pt-1)
	4		3,226%	= (Pt - Pt-1 + **Dt-1,t**) / Pt-1
			-1,316%	= (Pt - Pt-1) / Pt-1

Exercise A2_5:

The correlation between stocks Zo and Xp is 0.1. Risk and expected return of these stocks are:

Stock	Zo	Xp
Expected return (in % p.a.)	10	18
Standard deviation (in % p.a.)	15	30

(a) Find the portfolio weights for Zo and Xp that generate minimum portfolio risk.
(b) What is the minimum portfolio risk (i.e. standard deviation)?
(c) What is the expected return of this portfolio?

$p_{Z0,Xp} = 0.1$

(a)

$$x_A = \frac{\sigma_B^2 - \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \cdot \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}$$

$x_B = 1 - x_A$ →

Input:

$$\frac{0.3^2 - 0.1 \times 0.15 \times 0.3}{0.15^2 + 0.3^2 - 2 \times 0.1 \times 0.15 \times 0.3}$$

Result:

0.826086956521739130434782608695652173913043478260869565217...

x_A= 82.61%
x_B = 1-0.8261 = 0.1739 = 17.39%

b)

$$\sigma_P = \sqrt{x_A^2 \cdot \sigma_A^2 + x_B^2 \cdot \sigma_B^2 + 2 \cdot x_A \cdot x_B \cdot \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}$$ →

Input:

$$\sqrt{0.8261^2 \times 0.15^2 + 0.1739^2 \times 0.3^2 + 2 \times 0.8261 \times 0.1739 \times 0.1 \times 0.15 \times 0.3}$$

Result:

0.139175...

c)

$$E[R_P] = \sum_{i=1}^{N} x_i \cdot E[R_i]$$ →

Input:

$0.8261 \times 0.1 + 0.1739 \times 0.18$

Result:

0.113912

Exercise A2_6:

Mrs. P. invested 60% in stock A and the remainder in stock B. She builds her portfolio based on the following characteristics:

Stock	A	B
Expected return (in % p.a.)	15	20
Standard deviation (in % p.a.)	20	22
Corr (R_A,R_B)	0.5	

(a) Calculate the expected return and the standard deviation of her portfolio.
(b) How does the answer in (a) change, when the correlation coefficient changes to 0.0 or -0.5?
(c) Calculate the minimum variance portfolio based on the correlation in the table.
(d) Assume Mrs. P. <u>only</u> can invest in stock A or in stock B. Would it then be better or worse for her to only invest in stock A or only in stock B?

a)

$$E[R_P] = \sum_{i=1}^{N} x_i \cdot E[R_i]$$

0.6*0.15 + 0.4*0.2 = 0.17 = 17%

$$\sigma_P = \sqrt{x_A^2 \cdot \sigma_A^2 + x_B^2 \cdot \sigma_B^2 + 2 \cdot x_A \cdot x_B \cdot \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}$$

x_a= 0.6 = 60% (erster Satz aus der Angabe)

x_b=1-x1=0.4=40%

Input:

$$\sqrt{0.6^2 \times 0.2^2 + 0.4^2 \times 0.22^2 + 2 \times 0.6 \times 0.4 \times 0.5 \times 0.2 \times 0.22}$$

Result:

0.180842... = σ_P

b) Gleiches wie a) nur mit $p_{A,B}$ = 0.0 = -0.5

c)

$$x_A = \frac{\sigma_B^2 - \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \cdot \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}$$

$x_B = 1 - x_A$

Input:

$$\frac{0.22^2 - 0.5 \times 0.2 \times 0.22}{0.2^2 + 0.22^2 - 2 \times 0.5 \times 0.2 \times 0.22}$$

Result:

→ x_a= 0.594594594594594594594594594594594594594594594594594594594...

x_b=1-x1 = 40.54%

d) This depends on her risk preferences (risk aversion level).

Exercise A2_7:

An investor wants to invest into two companies with the following characteristics:

Stock A: Expected return = 18% p.a., volatility (σ) = 40% p.a.

Stock B: Expected return = 12% p.a., volatility (σ) = 25% p.a.

(a) Calculate the Minimum-Variance-Portfolio (weights, expected return, volatility), when the correlation between A and B amounts to 0.3.

(b) How does the answer in (a) change, when the correlation between the two stocks is -1?

a)

$$x_A = \frac{\sigma_B^2 - \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}{\sigma_A^2 + \sigma_B^2 - 2 \cdot \rho_{A,B} \cdot \sigma_A \cdot \sigma_B}$$

$x_B = 1 - x_A$

$$\sigma_P = \sqrt{x_A^2 \cdot \sigma_A^2 + x_B^2 \cdot \sigma_B^2 + 2 \cdot x_A \cdot x_B \cdot \rho_{A,B} \cdot \sigma_A \cdot \sigma_B} \qquad E[R_P] = \sum_{i=1}^{N} x_i \cdot E[R_i]$$

$$\frac{0.25^2 - 0.3 \times 0.4 \times 0.25}{0.4^2 + 0.25^2 - 2 \times 0.3 \times 0.4 \times 0.25}$$ $x_a = 0.2 = 20\% \rightarrow x_b = 1\text{-}0.2 = 80\%$

$$\sqrt{0.2^2 \times 0.4^2 + 0.8^2 \times 0.25^2 + 2 \times 0.2 \times 0.8 \times 0.3 \times 0.4 \times 0.25} = 0.236643 = 23.66\%$$

$$0.2 \times 0.18 + 0.8 \times 0.12 = 0.132 = 13.2\%$$

b)

$$\frac{0.25^2 - -0.4 \times 0.25}{0.4^2 + 0.25^2 - 2 \times (-1) \times 0.4 \times 0.25}$$ $x_a = 0.384615 = 38.46\% \rightarrow$

$x_b = 0.6154 = 61.54\%$

$$\sqrt{0.3846^2 \times 0.4^2 + 0.6154^2 \times 0.25^2 + 2 \times 0.3846 \times 0.6154 \times (-1) \times 0.4 \times 0.25} = 0.00001 \rightarrow = 0$$

$$0.3846 \times 0.18 + 0.6154 \times 0.12 = 0.1431 = 14.31\%$$

Exercise A2_8:

Assume that the expected rate of return of the market is 10% p.a. and the return on T-Bills (risk-free rate) is 1% p.a. The risk (standard deviation) of the market is 18% p.a.

(a) What is the equation of the capital market line (CML) for a portfolio P on the CML?

(b) If an expected return of 4% p.a. is desired, what is the risk (standard deviation) of this portfolio?

(c) If you have €1,000 to invest, how should you allocate it to achieve position in (b)?

(d) If you invest €300 in the risk-free asset and €700 in the market portfolio, how much money should you expect to have at the end of the year?

Risk = standard deviation

RF = 1%, E[Rm] = 10%, Risk = 18%

$$E[R_P] = R_F + \frac{E[R_m] - R_F}{\sigma_m} \cdot \sigma_P$$

a) E = 1% + (10-1)/18 * portfolio-risk

$E[R_P] = 1\% + 0.5 * \sigma_P$

b) portfolio return E = 4%, portfolio-risk?

Input:

$0.04 = 0.01 + 0.5\, r$

Solution:

$r \approx 0.06$

c) E = 4%, RF = 1%, E[Rd] = 10%

$E[R_P] = (1 - x_D) \cdot R_F + x_D \cdot E[R_D]$ => Auflösen nach $X_D = 1/3$ … X_D = weight of risky asset D

Now we invest 1000 € given the weight $X_D = 1/3$:

=> invest 2/3 (€666.67) risk free, and 1/3 (333.33) in the market

d) X_D given with 3/10, we now invest 1000€ and calculate the return

Input:

$300 \times 0.01 + 700 \times 0.1 + 1000$

Result:

1073

Exercise A2_9

The following information is available for three stocks and the market portfolio:

	Market Portfolio	Stock A	Stock B	Stock C
E[R] (% p.a.)	8	5	15	10
σ (% p.a.)	15	20	30	40
$\rho_{i,M}$	-	0.7	0.5	0.8

$\rho_{i,M}$ Correlation between stock i and the market portfolio

$R_F = 4\%$ p.a.

Question: Are the three stocks under- or overvalued (based on the CAPM)?

A2_9								
RF	4%							
	M	Stock A	Stock B	Stock C	Calculation			
E[R] (% p.a.)	8%	5%	15%	10%				
risk	15%	20%	30%	40%				
corr i,M		0,7	0,5	0,8				
cov i,M		0,021	0,0225	0,048	= corr i,M * risk i * risk M			
bi (cov/risk^2)		0,93333333	1	2,13333333				
SML		7,733%	8,000%	12,533%	E[Rp] = RF+ b(E[Rm] -RF)			
E[R] (% p.a.)		5%	15%	10%				
		overvalued	**undervalued**	**overvalued**	SML < E[R] -> undervalued; SML > E[R] -> overvalued			

Valuation is based in the SML, which is why we need the betas!

Exercise A2_10:

An investor wants to generate a portfolio with an expected return of 9% p.a. 2 investment possibilities exist: (i) stock market (expected return of the market portfolio = 11% p.a., volatility = 25 p.a.) and (ii) Money Market (3M-Euribor rate = 3% p.a.).

(a) Which portfolio should the investor choose if the CAPM is valid?

(b) Which risk (volatility) does this portfolio have?

(c) What is the beta of this portfolio?

(d) What is the expected return of this portfolio, when the stock market is expected to drop by 10%?

a)

$$E[R_P] = (1 - x_D) \cdot R_F + x_D \cdot E[R_D]$$

Input:

$$0.09 = (1 - w) \times 0.03 + w \times 0.11$$

Solution:

$$w \approx 0.75$$

X_D = 0.75

b) Portfolio risk (Only risky asset D is needed for the calculation)

X_D * $risk_D$= 18,75%

c)

$$\beta_P = \sum x_i \beta_i$$

$beta_{MarketPortfolio}$ = 1

$beta_{RiskFreeAsset}$ = 0

Input:

$$0.75 \times 1 + 0.25 \times 0$$

Result:

0.75

d) R[m]: 11% -> 1%

Calculation with $E[R_D]$ = -10%

$$E[R_P] = (1 - x_D) \cdot R_F + x_D \cdot E[R_D]$$

Input:

$$(1 - 0.75) \times 0.03 + 0.75 \times (-0.1)$$

Result:

$$-0.0675$$

in case you a desperately trying to study this shit, here is a motivational picture:

"We find it helps our less motivated employees.'