Knowledge Based Systems, 4.0 VU, 184.730

Exercise Sheet 3 – Answer Set Programming and Probabilistic Reasoning

This exercise sheet serves as a **preparation for the mandatory exercise test**, which covers the exercises and related background knowledge. You do *not* need to submit solutions.

For questions regarding exercises please visit the tutoring sessions (times are announced in **TUWEL**). You can find them in Conference room Hahn (room HG 03 06), Favoritenstr. 9-11, Stiege 3, 3rd floor, Institute of Information Systems. For questions of general interest, please use the **TISS** Forum or contact us on `kbsci-2018s@kr.tuwien.ac.at`.

Exercise 3.1: Consider the following normal logic program P, where a and b are constants, and X is a variable.

$$\begin{aligned} p(a)&.\\ p(b)&.\\ q(X) &\leftarrow p(X), \text{not } r(X).\\ r(X) &\leftarrow p(X), \text{not } q(X).\\ s &\leftarrow q(X).\\ &\leftarrow \text{not } s. \end{aligned}$$

- Compute the grounding $grnd(P)$ of the program P.
- Decide in a formal way, i.e., by calculating the *Gelfond-Lifschitz reduct*, whether the interpretation $I = \{p(a),\ p(b),\ r(a),\ q(b),\ s\}$ is a stable model of P.

Exercise 3.2: Show whether the following normal logic programs are stratified.

a) $P_1 = \{s(a) \leftarrow r(a).\ r(a) \leftarrow p(a), \text{not } q(a).\}$

b) $P_2 = \{p(a) \leftarrow \text{not } q(a).\ q(a) \leftarrow \text{not } p(a).\}$

Exercise 3.3: For a program P, we denote by $AS(P)$ the set of all answer sets of P. Let P, Q be programs. We say that P, Q are

(i) *equivalent* if $AS(P) = AS(Q)$, and

(ii) *strongly equivalent* if $AS(P \cup R) = AS(Q \cup R)$ for every program R.

Prove or refute if

1. whenever (ii) holds then also (i) and
2. the converse holds, i.e., whether (i) implies (ii).

Exercise 3.4: The combinatorial graph problem *dominating set* is defined as follows:

INSTANCE: Given a graph $G = (V, E)$ and a positive integer $k \leq |V|$.

QUESTION: Does there exist a subset $D \subseteq V$ such that $|D| \leq k$ and such that every vertex is either in D or adjacent to some vertex in D.

Define all required predicates, rules, and constraints to represent the *dominating set* problem as an answer-set program P such that

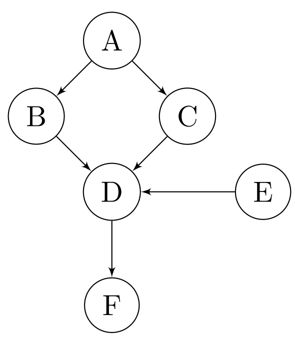
- each vertex $v \in V$ of G is denoted by a fact `vertex(v)`,
- each edge $(u, v) \in E$ of G is denoted by a fact `edge(u, v)`,
- the integer k is given by a single fact `size(k)`, and
- answer sets of P correspond to solutions D.

Exercise 3.5: Let S_1, S_2 be answer sets of an extended logic program P. Show that $S_1 \subseteq S_2$ implies $S_1 = S_2$.

Exercise 3.6: Let P be a normal logic program which consists of only n different ground atoms. What is the maximal amount of answer sets such a program can produce?

Hint: You may want to use *Sperner's Theorem*[1], which describes the largest possible families of finite sets none of which contain any other sets in the family.

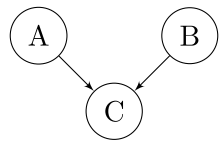
Exercise 3.7: Consider the following Bayesian Network:



1. Check whether the following (conditional) independencies hold:
   (a) $A \perp\!\!\!\perp_P E$
   (b) $A \perp\!\!\!\perp_P F \mid B$
   (c) $B \perp\!\!\!\perp_P E \mid F$
   (d) $B \perp\!\!\!\perp_P C \mid A$
   (e) $A \perp\!\!\!\perp_P D \mid BC$
2. Evaluate $P(a, b, \bar{c}, d)$, given the following conditional probabilities: $P(a) = \frac{3}{4}$, $P(b|a) = \frac{1}{3}$, $P(c|a) = \frac{1}{3}$, $P(e) = \frac{2}{3}$, $P(d|b, \bar{c}, e) = \frac{3}{4}$, $P(d|b, \bar{c}, \bar{e}) = \frac{1}{2}$ (other probabilities are not relevant).

Exercise 3.8: Consider the following Bayesian Network:

[1] See `https://en.wikipedia.org/wiki/Sperner%27s_theorem`



Find a probability distribution which shows that $A \perp\!\!\!\perp_P B \mid C$ does not hold in general on this network.

Note: this task shows that the third condition of d-separation (concerning a node with both edges incoming) is indeed necessary.