Homework 2 for the course Computability Theory

Deadline: May 3, 2021

1. Show that every infinite c.e. set contains an infinite computable subset.

2. Prove that there exists a simple set S containing all odd numbers. Use this to show that the union of two simple sets does not have to be simple.

3. a) Consider the collection $\mathcal{K}$ of all partial computable functions $f : \omega \to \omega$ that output 0 for at least two x's. Is the index set $\{i : \varphi_i \in \mathcal{K}\}$ of this collection computable? Is it c.e.? Explain why.

   b) Show that the index set $Comp = \{i : W_i \text{ is computable}\}$ is a Σ^0_3 set.

4. Prove that K and $\overline{K}$ are incomparable with respect to m-reducibility.

5. A is 1-*reducible* to B (notation: $A \leq_1 B$) if there is an injective computable function f such that, for all x,

$$x \in A \Leftrightarrow f(x) \in B.$$

   The 1-*degrees* are defined similarly to the m-degrees and the Turing degrees.

   a) Describe the 1-degrees containing computable sets.

   b) Let A be the set of even numbers. Is there a noncomputable set B such that $A \not\leq_1 B$?

6. Define the ω-*jump* of A as follows:

$$A^{(\omega)} = \{\langle m, n\rangle : m \in A^{(n)}\}.$$

   Show that:

   a) $A^{(n)} \leq_T A^{(\omega)}$ for all $n \in \omega$.

   b) $A^{(\omega)} \not\leq_T A^{(n)}$ for any $n \in \omega$.