Examination for "Logic and Computability"

May 30, 2017 — 3rd Exam for WS16/17

Matrikelnummer	FAMILY NAME	First Name

Task 1:
Prove in sequent calculus

$$(A \to \exists x B(x)) \to \exists x (A \to B(x))$$

where x is not free in A

Task 2:
Formalize the sentence

every number greater than 1 is divisible by a prime number

using the symbols $=$ (equality predicate), $<$ (predicate strictly less than), 1 (constant 1) and the binary predicate $|$ (divisible). [Note that you dont have a predicate prime number at your disposal]

Task 3:
Is the following set

$$\{x \mid \neg\exists y \Phi_x(y)\downarrow\}$$

Recursive, r.e. or none of them? (Motivate your answer)

Task 4:
Let I be any set of indexes of computable functions. Consider the following statement:

I is infinite if and only if I is extensional.

Is the statement true? (Motivate your answer and argue separately for each of the two cases: "if" and "only if".)

Task 5:
Prove or refute: $(\neg B \vee \Diamond A) \supset (B \supset A)$ characterizes reflexivity of Kripke frames.
Treat the two directions of the claim separately and argue directly about the corresponding frames.

Task 6:
Compute all factors of the clause $p(x, f(x)) \vee p(f(y), y) \vee p(f(a), z)$.
Specify the MGU and the unified literals for each factor. (x, y, z are variables, a is a constant.)

Task 7:
Show: The diagonal set U^* of the set U of Gödel numbers of all expressions that are not true sentences ($\notin \mathcal{T}$) is not expressible in any arithmetic system.