1	2	3	4	5	Σ	Grade

Examination for "Logic and Reasoning in Computer Science"

June 26th, 2024 **1st Exam for SS 2024**

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This exam sheet consists of five problems, yielding a total of 100 points. Good luck!

Problem 1. (25 points) Consider the formula :

$$(\neg q \land r) \land \neg((p \leftrightarrow r) \lor (p \to r))$$

(a) Which atoms are pure in the above formula?

(b) Compute a clausal normal form C of the above formula by applying the CNF transformation algorithm with naming and optimization based on polarities of subformulas;

(c) Decide the satisfiability of the computed CNF formula C by applying the DPLL method to C. If C is satisfiable, give an interpretation which satisfies it.

Problem 2. (20 points) Formalize the following arguments and verify whether they are correct:

1. I must be punished only if I am guilty; I'm guilty. Thus I must be punished
2. If I'm guilty, I must be punished; I'm not guilty. Thus, I must not be punished.

Note that verifying whether an argument is correct means to prove a statement of the form: from the hypothesis P_1 and .. and P_n, the conclusion Q follows (that is, you need to either formally prove $P_1, \ldots, P_n \models Q$, or exhibit a counterexample for the statement).

Problem 3. (20 points) Let A be a propositional, well-formed formula using $n \geq 1$ propositional variables such that

- A is not a propositional atom, and
- A is built from propositional atoms using only $\neg$ and $\leftrightarrow$.

How many branches does a splitting tree of A have? Provide a sufficiently detailed explanation of your answer.

Problem 4. (10 points) Provide either a tableau proof or a counterexample for the statement

$$\neg\exists x A(x) \models \exists x \neg(A(x) \lor A(f(x)))$$

If you provide a counterexample, you have to show that it is in fact a counterexample.

Problem 5. (25 points) Consider the formula:

$$a = b - 4 \land f(b+1) = c \land (f(a+5) \neq c \lor \mathit{read}(\mathit{write}(A, a+2, 3), b-3) = 1)$$

where b, c are constants, f is a unary function symbols, A is an array constant, $read, write$ are interpreted in the array theory, and $+, -, 1, 2, 3, ...$ are interpreted in the standard way over the integers.

Use the Nelson-Oppen decision procedure for reasoning in the combination of the theories of arrays, uninterpreted functions, and linear integer arithmetic. Use the decision procedures for the theory of arrays and the theory of uninterpreted functions and use simple mathematical reasoning for deriving new equalities among the constants in the theory of linear integer arithmetic. If the formula is satisfiable, give an interpretation that satisfies the formula.