Prüfung 5.2.2013

1. $f(x) = \frac{10x+3}{e^{2x}} - 1$

$f'(x) = 10x \cdot e^{-2x} + e^{-2x} \cdot 3 - 1$

$10 \cdot e^{-2x} + 10x \cdot e^{-2x} \cdot -2 - 3e^{-2x} \cdot -2$

$e^{-2x}(10 - 20x - 6) = \frac{-20x+4}{e^{2x}} = \frac{5x-1}{e^{2x}} \cdot -4$

$f''(x) = e^{-2x} \cdot 10 - 20x \cdot e^{-2x} - 6e^{-2x} = 10e^{-2x} \cdot (-2) - (20 \cdot e^{-2x} - 20x \cdot e^{-2x} \cdot -2) - 6e^{-2x} \cdot (-2)$

$= e^{-2x}(-20 - 20 + 20x + 12) = \frac{20x - 28}{e^{2x}} = \frac{5x-7}{e^{2x}} \cdot 4$

$f''(\frac{1}{5}) = \frac{1-7}{e^{\frac{2}{5}}} \cdot 4 = -16.09 < 0$ Maximum

$f'(x) = 0$

$-20x + 4 = 0$

$x = \frac{4}{20}$ $\quad x = \frac{1}{5}$

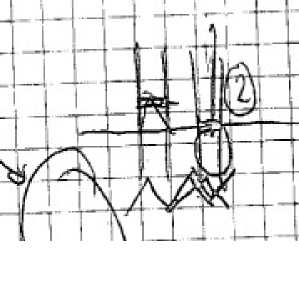
$f'(x) > 0$? $\quad \frac{-20x+4}{e^{2x}} > 0 \mid \cdot e^{2x}$

$-20x + 4 > 0 \mid -4$

[illegible] $20x < 4 \mid : 20$

$x < \frac{1}{5}$ umgedreht fallend

$\lim_{x \to 0} \frac{10x+3}{e^{2x}} - 1 = \frac{10 \cdot 0 + 3}{e^{2 \cdot 0}} - 1 = 3 - 1 = 2$



$\lim_{x \to \infty} \frac{10x+3}{e^{2x}} - 1 \overset{L'H}{=} \frac{\cancel{10}^{5}}{e^{2x} \cdot \cancel{2}} - 1 = -1$

Intervall [1;2] durch probieren

$NST: f:[1,2] \sim f(1) > 0, f(2) < 0 \Rightarrow f(c) = 0, c \in [1,2]$

Bisektion: $\frac{1+2}{2}$ $a=1, b=\frac{3}{2}$

$a=\frac{5}{4}, b=\frac{3}{2}$

$f(\frac{5}{4})>0$ $\frac{1+\frac{3}{2}}{2}=\frac{5}{4}$ $a=\frac{11}{8}, b=\frac{3}{2}$

$a=\frac{11}{8}, b=\frac{23}{16}$

$f(\frac{11}{8})>0$ $\frac{\frac{5}{4}+\frac{3}{2}}{2}=\frac{11}{8}$ $a=\frac{45}{32}, b=\frac{23}{16}$

$f(\frac{23}{16})<0$ $\frac{\frac{11}{8}+\frac{3}{2}}{2}=\frac{23}{16}$ $c=1.42=\frac{91}{64}$

$f(\frac{45}{32})>0$ $\frac{\frac{11}{8}+\frac{23}{16}}{2}=\frac{45}{32}$

$f(\frac{91}{64})=0.00$ $\frac{\frac{45}{32}+\frac{23}{16}}{2}=\frac{91}{64}$

2) $\iint_D \left(1+\frac{10x}{1+x^2}+y\right)dx\,dy$ $-1\le x\le 1, -1\le y\le 1$

$\int_{-1}^{1}\int_{-1}^{1}\left(1+\frac{10x}{1+x^2}+y\right)dx\,dy$

$\int_{-1}^{1}\int_{-1}^{1} 1\,dx\,dy+\int_{-1}^{1}\int_{-1}^{1}\frac{10x}{1+x^2}dx\,dy+\int_{-1}^{1}\int_{-1}^{1} y\,dx\,dy$

$\int_{-1}^{1} 2\,dy+\int_{-1}^{1} 2y\,dy+\int_{-1}^{1}\int_{-1}^{1}\frac{10x}{u}\frac{du}{2x}dx\,dy$ $u=1+x^2$ $\ln|1+x^2|$ $dx=\frac{du}{2x}$ $= 4+1-1+\cancel{\int_{-1}^{1}\ln 2-\ln 2}$

$=4$

3. $y'' - y' - 2y = -12x$

y_h: $y_h(x) = e^{\lambda x}$

$\lambda^2 e^{\lambda x} - \lambda e^{\lambda x} - 2e^{\lambda x} = 0 \mid : e^{\lambda x}$

$\lambda^2 - \lambda - 2 = 0$

$\lambda_{1,2} = \frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 2} = \frac{1}{2} \pm \sqrt{\frac{9}{4}} = \frac{1}{2} \pm \frac{3}{2}$

$\lambda_1 = -1,\ \lambda_2 = 2 \quad \lambda_1 \neq \lambda_2$ reell!

$y_h = C_1 \cdot e^{-x} + C_2 \cdot e^{2x}$

Versuchslösung

y_p: $-12x$ linear $\Rightarrow y_p = A_0 + A_1 x$, $y_p' = A_1$, $y_p'' = 0$

einsetzen in DGL

$0 - A_1 - 2(A_0 + A_1 x) = -12x$

$-A_1 - 2A_0 - 2A_1 x = -12x$ Vergleich der ~~Konst~~ Koeffizienten

1. $-2A_1 = -12x \Rightarrow A_1 = 6$ $\quad y(x) = y_h + y_p$

2. $-A_1 - 2A_0 = 0$

$-6 - 2A_0 = 0$

$A_0 = -3$

$y_p = -3 + 6x$ $\qquad \underline{y(x) = \frac{C_1}{e^x} + C_2 \cdot e^{2x} - 3 + 6x}$

3. Probe: $y(x) = \frac{C_1}{e^x} + C_2 \cdot e^{2x} - 3 + 6x$

$y'(x) = \frac{C_1}{e^x} + C_2 \cdot e^{2x} \cdot 2 + 6$

$y''(x) = \frac{C_1}{e^x} + 4C_2 \cdot e^{2x}$

Einsetzen in Angabe:

$\frac{C_1}{e^x} + 4C_2 \cdot e^{2x} - \left(\frac{C_1}{e^x} + C_2 e^{2x} \cdot 2 + 6\right) - 2\left(\frac{C_1}{e^x} + C_2 \cdot e^{2x} - 3 + 6x\right)$

$= -12x$?

$\cancel{2\frac{C_1}{e^x}} - \cancel{2\frac{C_1}{e^x}} + \cancel{2C_2 e^{2x}} - \cancel{2C_2 \cdot e^{2x}} - \cancel{6} + \cancel{6} - 12x = -12x$

$\underline{\underline{\text{w. A.}}}$