Examination for "Logic and Computability"

June 24th, 2021 **Last exam for WS 20/21**

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Task 1:
Consider the following reasoning:

(a) Every rigorous person admires some mathematician.

(b) Every mathematician admires some rigorous person.

(c) Alf admires only himself.

(d) Either Alf is not a mathematician or Alf is a rigorous person.

Formalize (a)-(d), and establish whether (d) is a consequence of (a)-(c).
(Provide a proof or a countermodel.)

Task 2:
Prove or disprove the following statements:

- $\models \exists x(\neg A(x) \vee \forall x A(x))$
- $(\exists x A(x) \to \exists x B(x)) \models \forall x(A(x) \to B(x))$

Task 3:
Are the sets

- $\{x \mid \exists y \Phi_x(y) \neq 5\}$
- $\{x \mid \Phi_x(0) \downarrow \wedge\, x \leq 5\}$

recursive, r.e. or none of them? Motivate your answers.

Task 4:
Consider the following statements:

1. If I is infinite then I is extensional.
2. I is extensional then I is infinite.

Are they true? (Motivate your answer)

Task 5:
Compute all factors of the clause $p(f(y), f(x)) \vee p(f(x), z) \vee p(y, z)$.

Task 6:
Let G be the modal formula $\Diamond\neg A \vee \Diamond\Box A$. Prove or refute:
(1) G is valid in every reflexive frame.
(2) If $\mathcal{F} \models G$ for a frame $\mathcal{F}$, then $\mathcal{F}$ is reflexive.

Task 7:
Use the proof of the ADRF theorem to show that the function $f(g(x))+g(f(x))+3$ is arithmetically definable, if f and g are arithmetically definable. *Hint:* Specify the witnessing formula, following slide 23 of the last set of slides (on incompleteness).