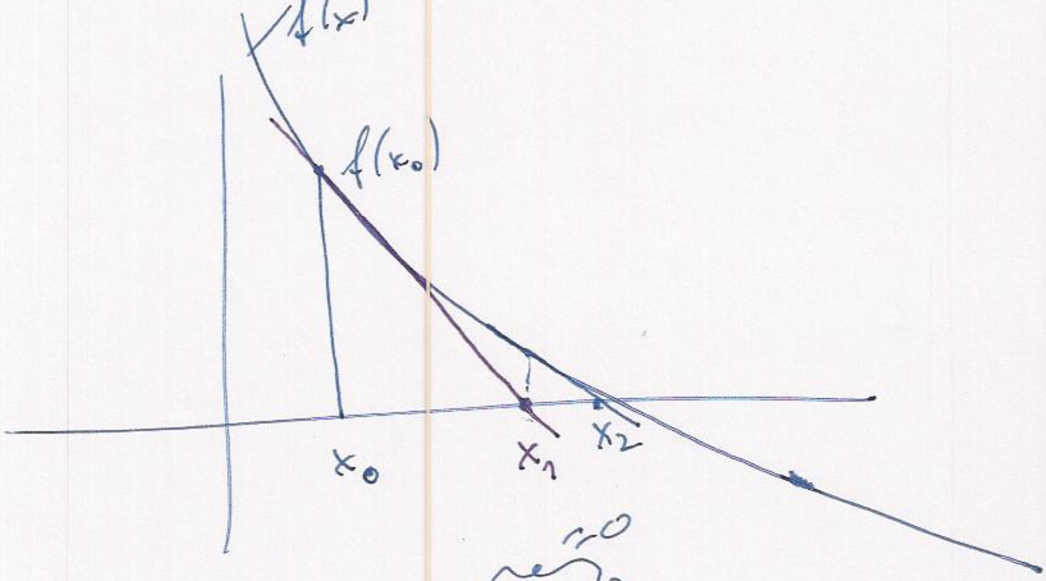
· Newton'sche Näherungsverfahren

Vor.: · $f: \overset{[a,b]}{\overset{\text{"}}{I}} \to \mathbb{R}$ zweimal stetig differenzierbar

· $f'(x) \neq 0$ für alle $x \in I$

Idee: betrachte Schnittpunkt der Tangente mit x-Achse

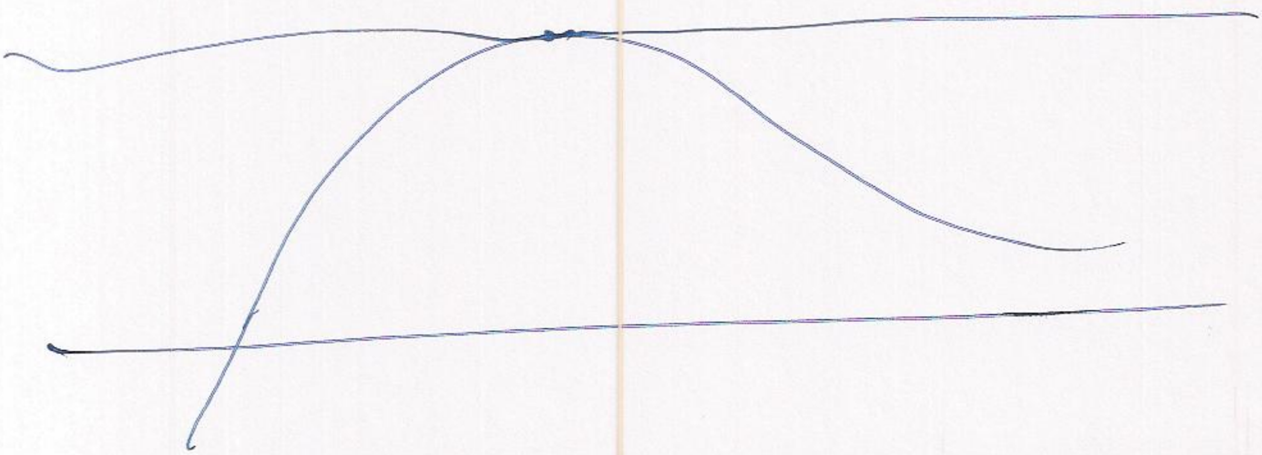


$$\frac{f(x_0) - \overbrace{f(x_1)}^{=0}}{x_0 - x_1} = f'(x_0)$$

$$\frac{x_0 - x_1}{f(x_0)} = \frac{1}{f'(x_0)}$$

$$x_0 - x_1 = \frac{f(x_0)}{f'(x_0)} \Rightarrow \boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



$\Rightarrow$ Iterationsfolge:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

es lässt sich zeigen: falls Startwert x_0
"genügend nahe" bei Nst. x^*

$\Rightarrow$ Folge $(x_n)_n$ konvergiert gegen x^*

Vorteil Newton sches Näherungsverf.:

- konvergiert "schneller" als regula falsi

: Konvergenzordnung $p = 2$ (einfache NST)

(Mehrfach NST $p = 1$)

Nachteil: benötigt Differenzierbarkeit der Fkt. f.

$$|x_{n+1} - x^*| \le M \cdot |x_n - x^*|^{p}$$

Bsp.: $f(x) = x^2 - a = 0$, $a > 0$ konstant

$$x^2 = a \Rightarrow x_{1,2} = \pm\sqrt{a}$$

Newton-Verf.:

$$f'(x) = 2x$$

$$\boxed{x_{n+1}} = x_n - \frac{f(x_n)}{f'(x_n)} =$$

$$= x_n - \frac{x_n^2 - a}{2 \cdot x_n} =$$

$$= \frac{2x_n^2 - x_n^2 + a}{2x_n} =$$

$$= \frac{x_n^2 + a}{2x_n} =$$

$$= \boxed{\frac{1}{2} \cdot \left(x_n + \frac{a}{x_n} \right)}, \quad x_0 > 0$$

Konv. $\Rightarrow$ $(x_n)_n \longrightarrow \sqrt{a}$

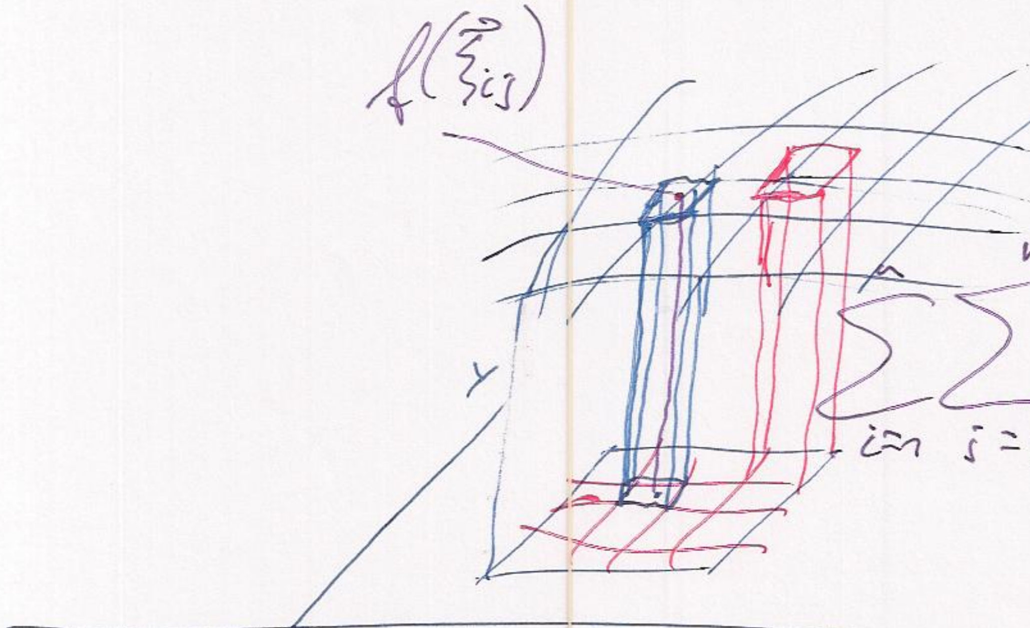
Bereichsintegrale (= Gebietsintegrale)

B ... zweidimensionaler Bereich

zuerst: $B = [a, b] \times [c, d]$

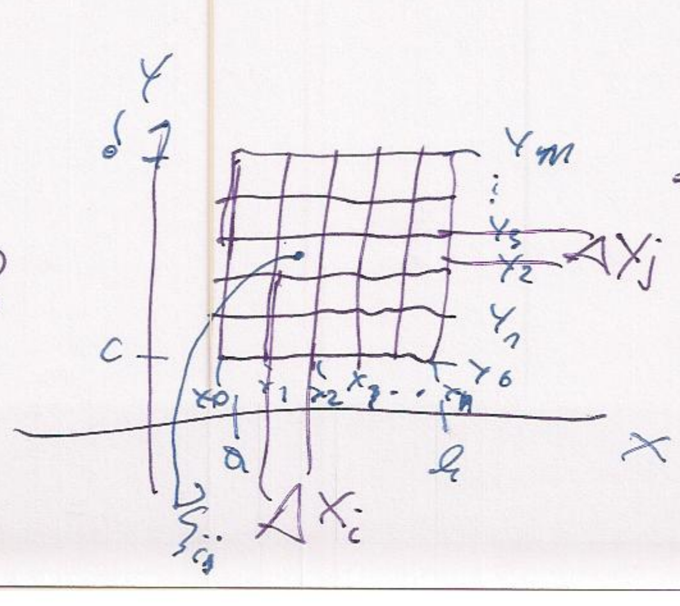
Rechtecksbereich

$f(x, y)$ ... <u>stetige Fkt.</u> in 2 Variablen

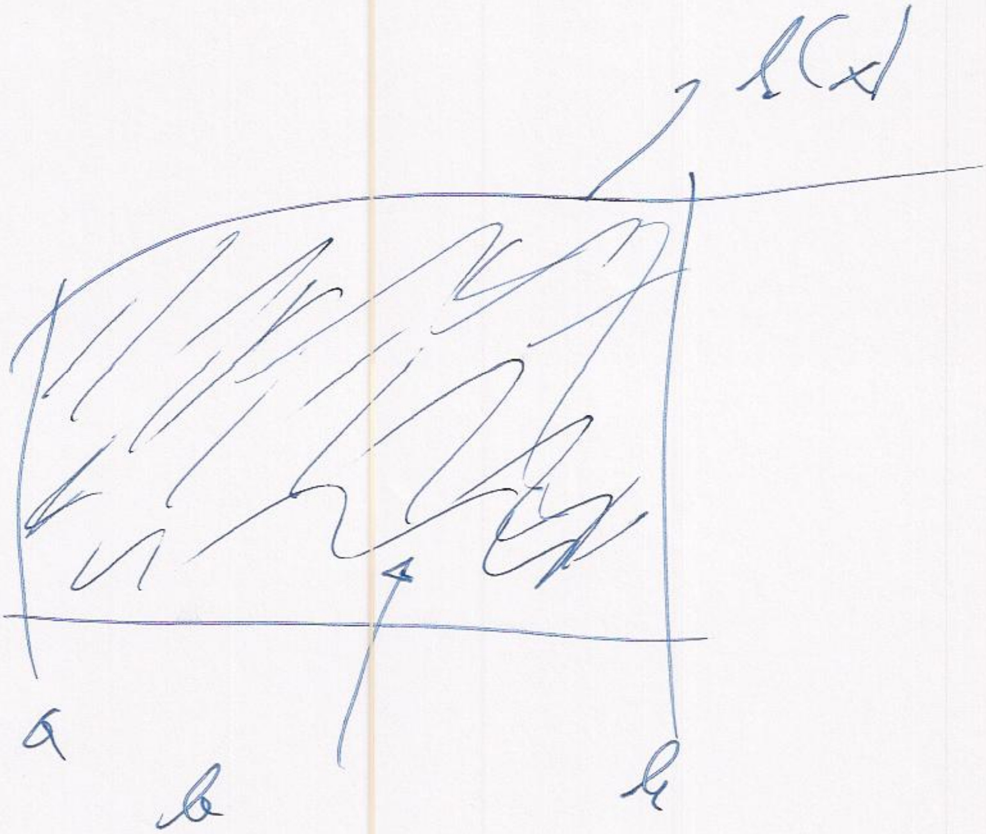


$$\sum_{i=1}^{n}\sum_{j=1}^{m} f(\vec{\xi}_{ij})\cdot\Delta x_i\,\Delta y_j \;\xrightarrow{\lim\limits_{n\to\infty}\lim\limits_{m\to\infty}}\; \iint_B f(x,y)\cdot dx\cdot dy$$

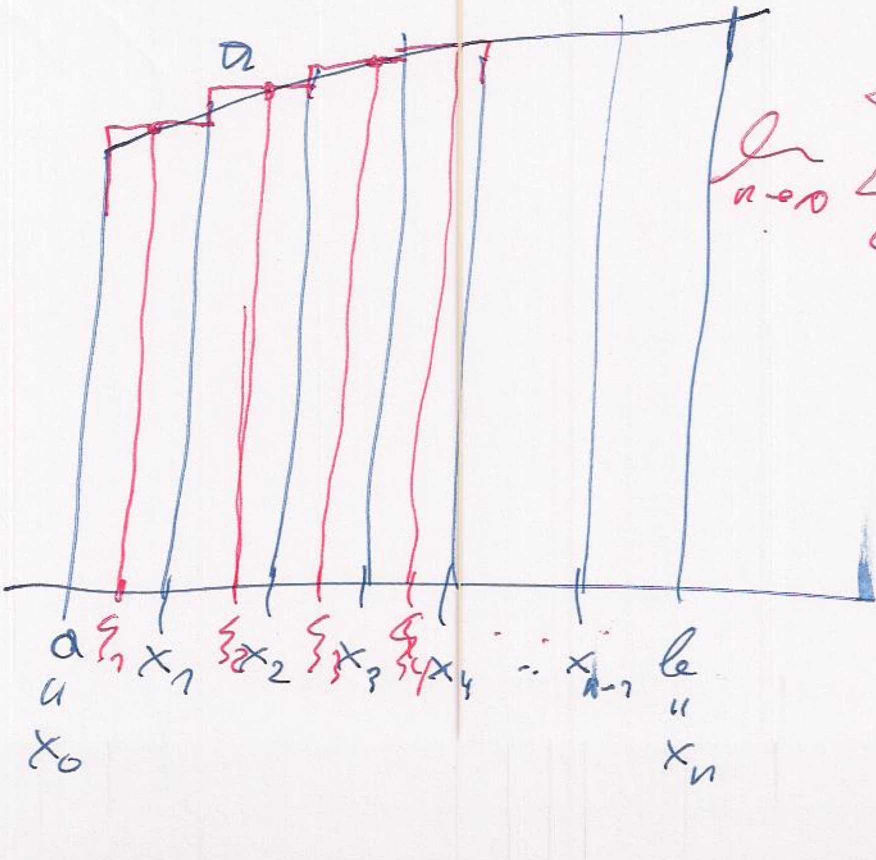
Bereich B



„Rechtecke aufstellen"



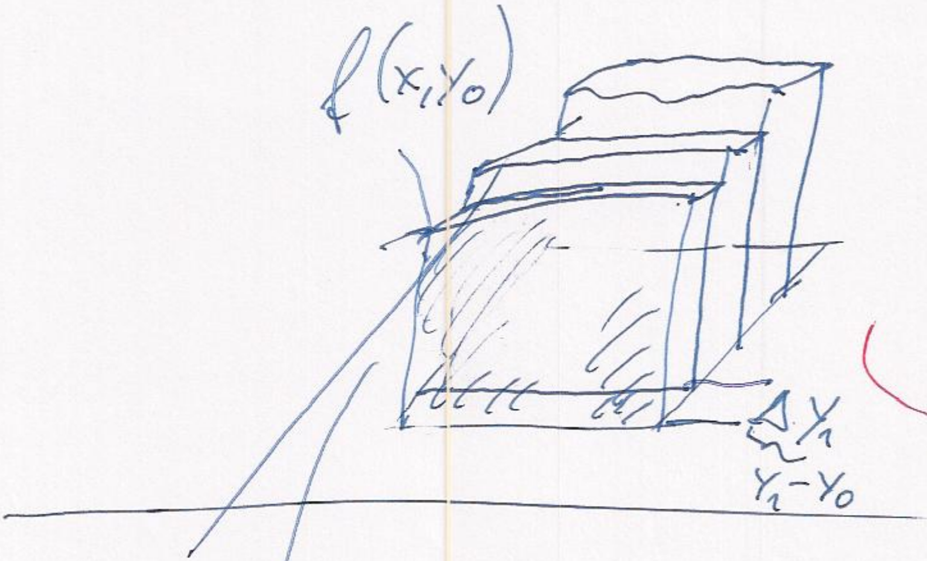
$$\int_a^b f(x)\cdot dx$$



$$\lim_{n\to\infty}\sum_{i=1}^{n} f(\xi_i)\cdot \underbrace{\Delta x_i}_{x_i - x_{i-1}}$$

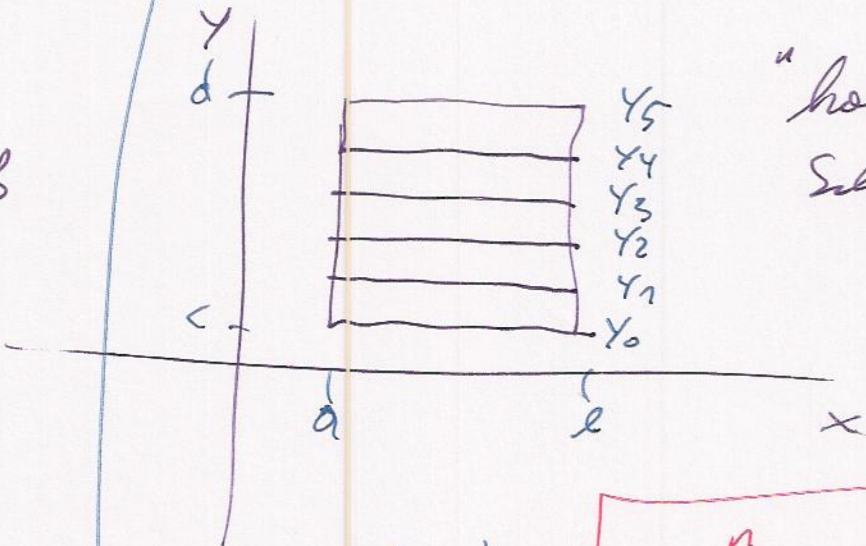
$$\| $$

$$\int_a^b f(x)\cdot dx$$



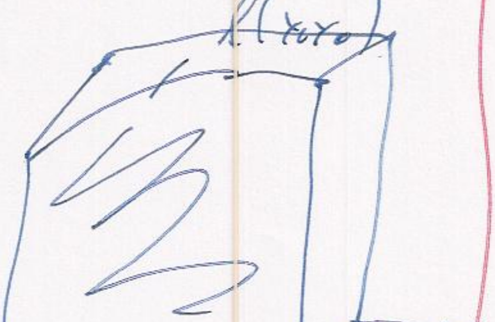
$f(x,y_i)\cdot\Delta Y_i$

Bereich B



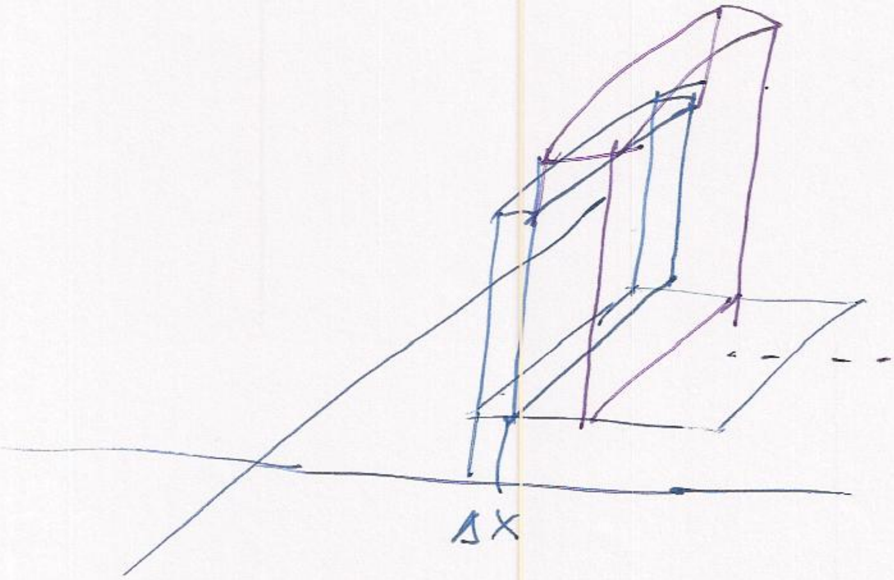
„horizontale Scheiben"

$$\sum_{i=1}^{n} \left(\int_a^b f(x,y_i)\cdot dx \right) \cdot \Delta Y_i$$

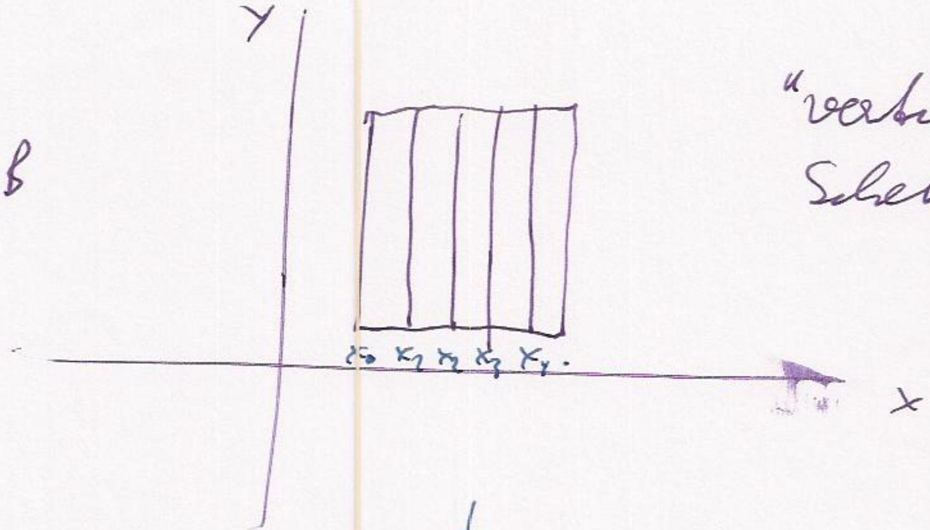


$$\left(\int_a^b f(x,y_0)\cdot dx \right) \cdot \Delta Y_1$$

$$\int_c^d \left(\int_a^b f(x,y)\cdot dx \right) \cdot dy$$



Bereich B



"vertikale Scheiben"

$$\sum_{i=1}^{n}\left(\int_{c}^{d} f(x_i, y)\cdot dy\right)\cdot \Delta x_i$$

$$\int_{a}^{b}\left(\int_{c}^{d} f(x, y)\cdot dy\right)\cdot dx$$

es gilt: egal wie aufgeteilt wird, das Ergebnis ist das selbe

$$B = [a, b] \times [c, d]$$

$$\Rightarrow \iint_B f(x,y) \cdot dx \cdot dy$$

$$\| $$

$$\int_c^d \left(\int_a^b f(x,y) \cdot dx \right) dy$$

$$\| $$

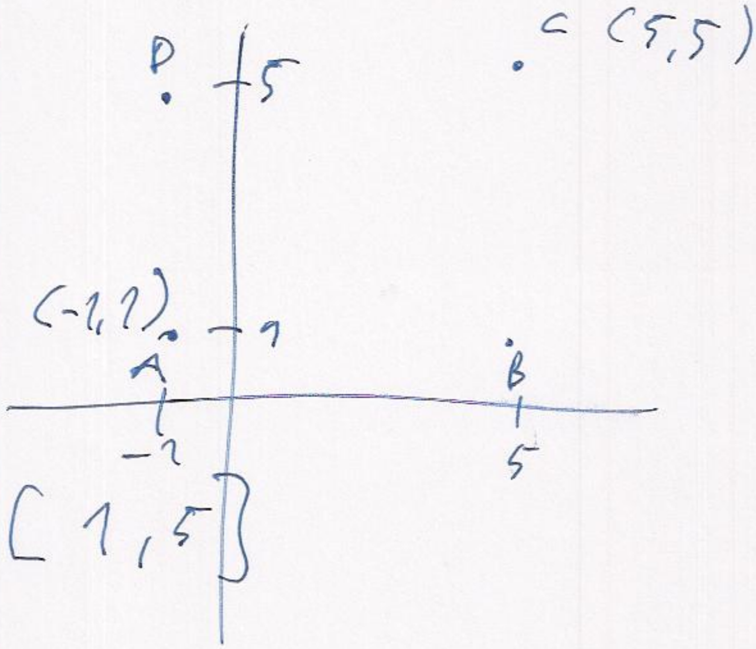
$$\int_a^b \left(\int_c^d f(x,y) \cdot dy \right) \cdot dx$$

6.39

$$\iint (xy + x^2 - y^2)\,dx\,dy$$

B

B F



$B = [-1,5] \times [1,5]$

$$\int_{1}^{5}\left(\int_{-1}^{5}(xy + x^2 - y^2)\cdot dx\right)\cdot dy =$$

$$= \int_{1}^{5}\left(\left(\frac{x^2}{2}y + \frac{x^3}{3} - y^2\cdot x\right)\Bigg|_{-1}^{5}\right)\cdot dy =$$

$$= \int_{1}^{5}\left(\frac{25}{2}y + \frac{125}{3} - 5y^2 - \frac{1}{2}y + \frac{1}{3}y^2\right)\cdot dy =$$

$=$

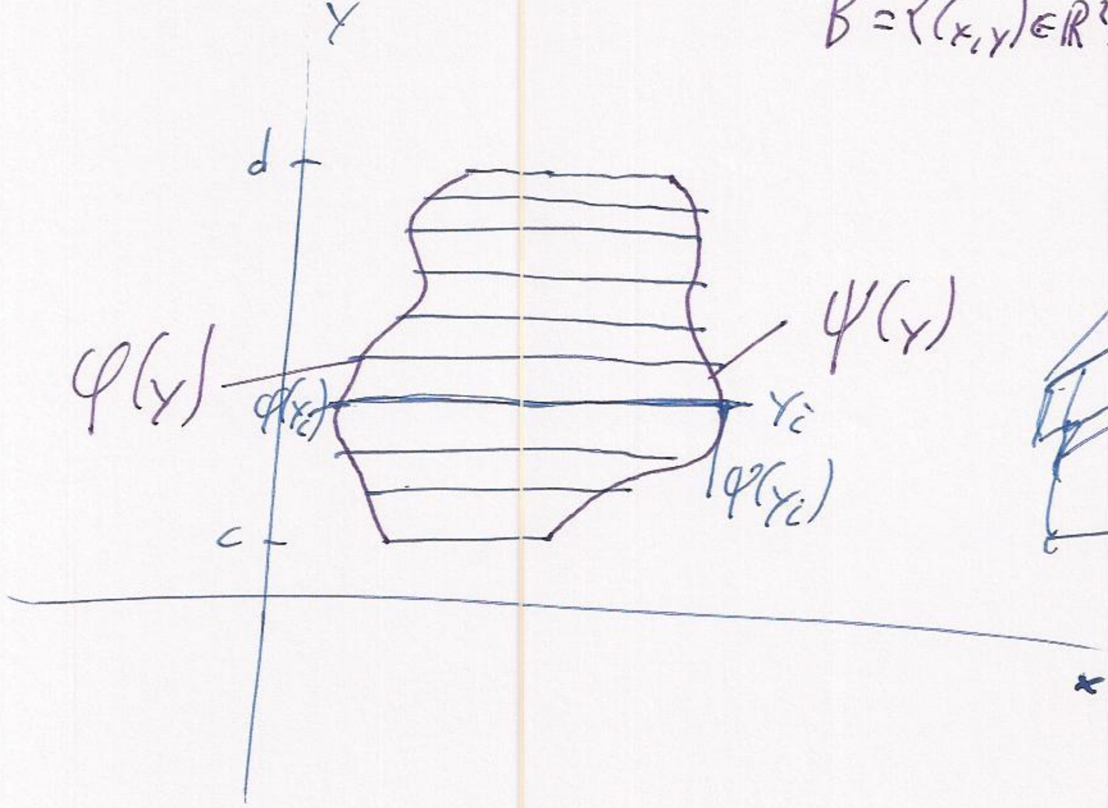
$$= \int_1^5 \left(12y + \overbrace{\frac{126}{3}}^{42} - 6y^2\right)\cdot dy =$$

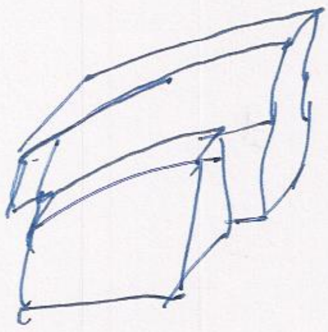
$$= 12\frac{y^2}{2} + 42y - 6\frac{y^3}{3} \Big|_1^5 =$$

$$= 6\cdot 25 + 42\cdot 5 - 2\cdot 125 - 6 - 42 + 2$$

allgemeinere Bereiche

$$B = \{(x,y) \in \mathbb{R}^2 : \varphi(y) \le x \le \psi(y),\; c \le y \le d\}$$





Satz von Fubini: Sei $B = \{(x,y) \in \mathbb{R}^2 : \varphi(y) \le x \le \psi(y),\; c \le y \le d\}$,

wobei $\varphi(y)$, $\psi(y)$ stetige Fkt. $\Rightarrow$

$$\iint_B f(x,y) \cdot dx \cdot dy = \int_c^d \left(\int_{\varphi(y)}^{\psi(y)} f(x,y) \cdot dx \right) dy$$